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Efficient Portfolio Rebalancing

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MANY INVESTORS WANT TO HOLD PORTFOLIOS combining assets in fixed proportions, e.g., 60 percent domestic equities, 10 percent international equities and 30 percent bonds. Once such a portfolio is formed, actual asset proportions will diverge from the target proportions as security prices fluctuate. Left free to drift, a portfolio can evolve into an asset mix with decidedly different risk and return characteristics than the target mix.

Rebalancing refers to the adjustment of a portfolio's asset weights towards the target weights. In the absence of trading costs, a portfolio could be rebalanced continuously and never deviate from its target weights. Given that trading is costly, continuous rebalancing is not feasible.[2] Procedures for determining *when* and *how* to rebalance are required. The "how" decision concerns whether to restore each asset to its precise target weight (*full adjustment*) or to move some or all assets part way to their targets (*partial adjustment*).

It is common practice to rebalance by making full adjustments at regular calendar intervals, e.g., quarterly or annually. Such calendar-determined rebalancing seldom occurs at the most opportune times. When markets are tranquil, quarterly rebalancing may be too frequent and costly. When markets are turbulent, annual rebalancing may be too infrequent permitting asset weights to drift far from their targets. Whatever the rebalancing frequency adopted, the amount of trading on each rebalancing date will be excessive if full adjustments to target weights are made and trading costs are proportional to the dollar value of assets traded.

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[2]Leland (1985) shows that an infinite amount of trading is required to keep assets continuously at their target proportions.

Efficient Rebalancing

An *efficient rebalancing strategy* trades only when the benefits of rebalancing exceed the costs. Rebalancing is beneficial because it improves *tracking accuracy*. Tracking accuracy is measured by *tracking error*.[3] A reduction in tracking error indicates improved tracking accuracy. The costs of rebalancing are the trading costs incurred. Trading costs are assumed to be proportional to the dollar value of the assets traded. (For example, if trading costs equal 50 basis points of the value of assets bought or sold, it costs $500 for rebalancing trades involving sales of $50,000 and purchases of $50,000.)

It is assumed that the trade-off between trading costs and tracking accuracy can be expressed by a *utility function*:

(1) $\text{Utility} = -\text{Trading Costs} - p\,(\text{Tracking Error})^2.$

The investor's objective in rebalancing decisions is to maximize utility. In the utility function, trading costs are expressed as fractions of portfolio value. (For a $1 million portfolio, trading costs of $500 represent 5 basis points of portfolio value.) The letter *p* represents the investor's price of tracking accuracy.

The *price of tracking accuracy* indicates roughly how much an investor is willing to pay for an additional basis point reduction in predicted tracking error. For an investor caring nothing about tracking accuracy, *p* is zero, and a policy of never rebalancing (so that trading costs are zero) maximizes utility. For an investor concerned about tracking accuracy, *p* is greater than zero. The greater the importance of tracking accuracy, the larger the value of *p*.

The requirement that rebalancing occurs only when its benefits exceed its costs creates a *Non-Trading Region* (NTR) about the target asset proportions as illustrated in figure 1. A target portfolio of 60 percent equities and 40 percent bonds is assumed. The boundaries of the NTR about the target equity ratio are 65 percent and 55 percent. At each boundary, the cost of the smallest possible rebalancing

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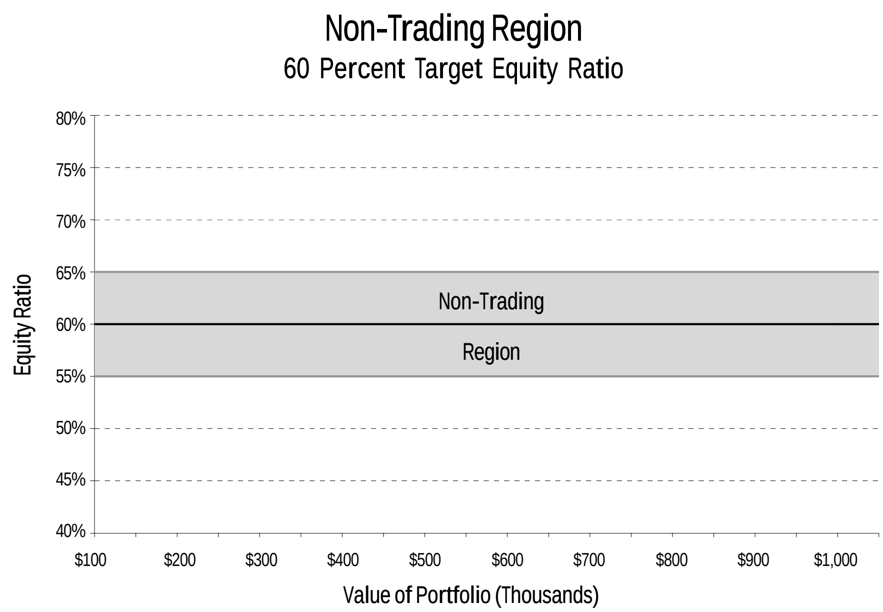
[3]Tracking accuracy refers to the correspondence between the returns of an investor's portfolio and the returns of a target portfolio or index. A portfolio tracks a target or index perfectly when every return of the portfolio exactly equals the return of the target or index. *Realized* Tracking Error (*TE*) is a measure of tracking accuracy:

$$TE = \sqrt{\frac{12}{T}\sum_{t=1}^{T}(R_{P,t} - R_t^*)^2}.$$

In this equation, $R_{P,t}$ is the return of portfolio *P* in month *t*, R_t^* is the return of the corresponding target portfolio in month *t*, *T* is the number of months in the sample period, and the sum of squares is multiplied by 12 to annualize the measure. *Predicted* tracking error squared is used in equation (1) to simplify the mathematics involved in solving for a maximum.

Figure 1



trade exactly equals the value of the resulting improvement in tracking accuracy — rebalancing is a break-even proposition. For equity ratios within the limits, the costs of rebalancing exceed the gains so rebalancing produces losses. For equity ratios outside the limits, the gains in tracking accuracy exceed the trading costs so it pays to rebalance — but only to the nearer limit of the NTR. Moving within the nearer boundary is unrewarding because, with costs exceeding gains within the NTR, the net benefits of rebalancing are reduced. (The efficient approach to rebalancing is described more fully in this paper's appendix.)

Simulation Experiments

The merits of conventional calendar-based methods and the efficient rebalancing approach are evaluated by simulation experiments. Three constant target mixes are specified (table 1). Each target mix is a combination of five asset-class portfolios. The three target mixes are identified by their equity ratios: high (80% equities), medium (70%) and low (60%). Accurate estimates of asset-class standard deviations and correlations (table 2) are available. The costs of trading each asset-class portfolio (table 3) are proportional to the value of the assets traded.

Table 1

Target Mixes

Asset Class	Equity Ratio High (80%)	Medium (70%)	Low (60%)
S&P 500	20%	25%	30%
DFA Large Value	20%	15%	10%
DFA 9-10	20%	15%	10%
EAFE	20%	15%	10%
Lehman Gov./Corp. Bonds	20%	30%	40%

Table 2

Estimated Standard Deviations and Correlations of Annual Excess Rates of Return

1970-1996

Asset Class	Standard Deviation	Correlations S&P 500	DFA L.V.	DFA 9-10	EAFE
S&P 500	16.5%				
DFA Large Value	17.1%	.873			
DFA 9-10	25.8%	.769	.800		
EAFE	23.2%	.547	.437	.431	
Lehman Gov./Corp. Bonds	7.9%	.540	.614	.299	.237

Table 3

One-Way Trading Costs

Asset Class	Costs
S&P 500	0.25%
DFA Large Value	0.50%
DFA 9-10	1.50%
EAFE	0.75%
Lehman Gov./Corp. Bonds	0.10%

The simulation experiments are conducted subject to several assumptions:

- The funds under management are tax exempt. The statistical results reported may be viewed as pre-tax measures for taxable funds. For taxable funds, capital-gains taxes significantly raise the costs of rebalancing.[4] The efficient rebalancing approach can be modified to incorporate capital-gains taxes, and it should then yield after-tax results that are highly superior to those of calendar-based or other rebalancing procedures that ignore these taxes.

- No cash inflows or outflows occur. All rebalancing operations require the sale of some assets and the purchase of others. This produces clear measures of the costs of different rebalancing approaches.

- Rebalancing transactions involve the sale and purchase of securities in open markets. The investor bears the full costs of these trades. The costs assumed (table 3) exceed the typical costs of exchanging the shares of one mutual fund for another within the same family of mutual funds.

- The five asset-class portfolios do not require internal rebalancing.

For each of the three target mixes, ten portfolios are maintained:

- A *target-mix portfolio* – This portfolio is rebalanced at no cost to precise target weights at the end of each month. It serves as a benchmark for the other portfolios. Tracking accuracy is measured relative to the returns of the target-mix portfolio.

- Five *calendar-rebalanced portfolios* – Each of these portfolios is adjusted fully to its target weights at a different frequency: *monthly*, *quarterly*, *semi-annually*, *annually* and *biennially*.

- Three *efficiently rebalanced portfolios* – Decision making for each of these portfolios is based on a different price of tracking accuracy: p=8, p=4 and p=2. The larger the value of p, the greater the importance of tracking accuracy relative to trading costs.

- A *buy-and-hold portfolio* – This portfolio begins by holding assets in their target weights, but it is never rebalanced. This portfolio provides an indication of the deterioration in tracking accuracy occurring in the absence of rebalancing over extended periods.

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[4]Rebalancing often involves the sale of assets that have appreciated in value resulting in the realization of a taxable capital gain. The capital-gains tax represents an additional cost of rebalancing. For example, if assets with a cost basis of $80,000 are sold for $100,000, the net proceeds from the sale are reduced by $4,000 (or 4 percent) if the $20,000 capital gain is subject to a 20 percent tax rate.

Table 4

High Equity Ratio Portfolios

Simulated Rebalancing with Historical Data
1971-1996

Rebalancing Strategy	Trades Per Year	Annual Trading Costs	Tracking Error	Realized Utility p=8	Realized Utility p=4	Realized Utility p=2	Turnover	Compound Return	Standard Deviation	Sharpe Ratio
Target Mix	12	0	0	0	0	0	12.14%	13.40%	14.30%	0.557
Monthly	12	0.17%	0.06%	-0.167%	-0.166%	-0.166%	12.21%	13.21%	14.28%	0.543
Quarterly	4	0.11%	0.22%	-0.112%	-0.110%	-0.109%	7.54%	13.33%	14.29%	0.552
Semiannually	2	0.08%	0.32%	-0.092%	-0.088%	-0.086%	5.74%	13.41%	14.36%	0.557
Annually	1	0.06%	0.50%	-0.080%	-0.070%	-0.064%	4.24%	13.42%	14.34%	0.556
Biennially	0.5	0.05%	0.83%	-0.108%	-0.081%	-0.067%	3.88%	13.82%	14.39%	0.582
Efficient (p=8)	4.04	0.04%	0.32%	-0.046%			4.20%	13.49%	14.31%	0.563
Efficient (p=4)	1.77	0.02%	0.60%		-0.038%		2.72%	13.64%	14.32%	0.574
Efficient (p=2)	0.31	0.01%	0.98%			-0.025%	0.81%	13.21%	14.02%	0.547
Buy & Hold	0	0	1.55%	-0.192%	-0.096%	-0.048%	0	13.36%	14.48%	0.536

Table 5

Medium Equity Ratio Portfolios

Simulated Rebalancing with Historical Data
1971-1996

Rebalancing Strategy	Trades Per Year	Annual Trading Costs	Tracking Error	Realized Utility p=8	Realized Utility p=4	Realized Utility p=2	Turnover	Compound Return	Standard Deviation	Sharpe Ratio
Target Mix	12	0	0	0	0	0	11.74%	12.75%	12.94%	0.550
Monthly	12	0.14%	0.05%	-0.139%	-0.139%	-0.139%	11.80%	12.59%	12.93%	0.537
Quarterly	4	0.09%	0.22%	-0.094%	-0.092%	-0.091%	7.19%	12.71%	12.93%	0.547
Semiannually	2	0.07%	0.33%	-0.078%	-0.074%	-0.071%	5.48%	12.77%	13.00%	0.551
Annually	1	0.05%	0.52%	-0.072%	-0.061%	-0.056%	4.08%	12.76%	12.97%	0.548
Biennially	0.5	0.04%	0.82%	-0.098%	-0.071%	-0.058%	3.66%	13.13%	12.98%	0.575
Efficient (p=8)	3.88	0.03%	0.31%	-0.038%			4.09%	12.87%	12.93%	0.560
Efficient (p=4)	1.50	0.02%	0.60%		-0.033%		2.53%	12.91%	12.94%	0.563
Efficient (p=2)	0.58	0.01%	0.91%			-0.023%	1.17%	12.62%	12.80%	0.541
Buy & Hold	0	0	1.74%	-0.241%	-0.121%	-0.060%	0	12.84%	13.32%	0.530

Historical Rebalancing Simulations, 1971-96

The initial simulations apply the alternative rebalancing strategies to the actual 1971-96 history of monthly returns. Rebalancing summary measures for each of the three target mixes are reported in tables 4-6. In each table, the first column

Table 6

Low Equity Ratio Portfolios

Simulated Rebalancing with Historical Data, 1971-1996

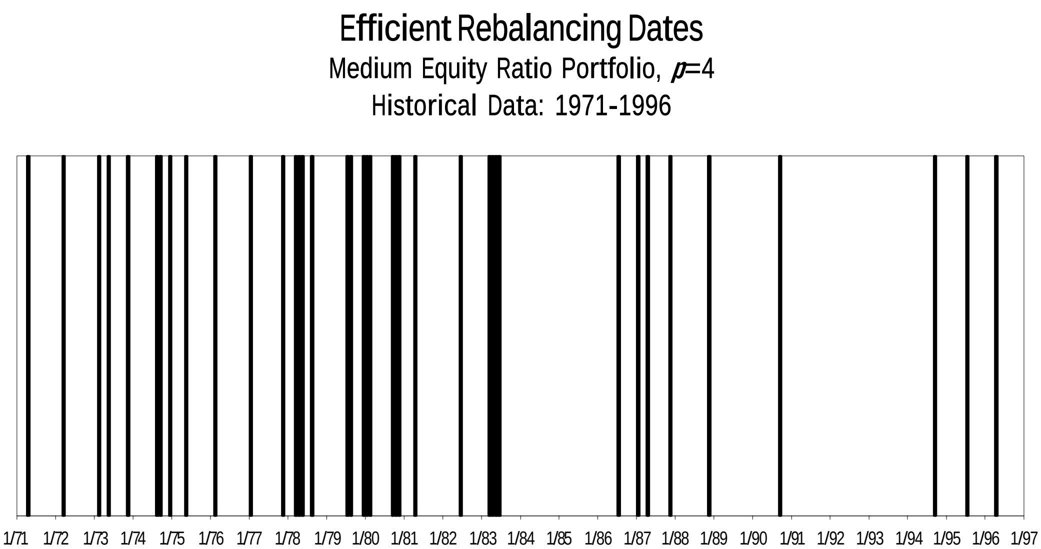
Rebalancing Strategy	Trades Per Year	Annual Trading Costs	Tracking Error	Realized Utility $p=8$	Realized Utility $p=4$	Realized Utility $p=2$	Turnover	Compound Return	Standard Deviation	Sharpe Ratio
Target Mix	12	0	0	0	0	0	11.00%	12.07%	11.72%	0.536
Monthly	12	0.11%	0.04%	-0.108%	-0.108%	-0.108%	11.06%	11.95%	11.71%	0.525
Quarterly	4	0.07%	0.22%	-0.074%	-0.072%	-0.071%	6.65%	12.05%	11.71%	0.534
Semiannually	2	0.05%	0.33%	-0.062%	-0.058%	-0.055%	5.08%	12.09%	11.78%	0.538
Annually	1	0.04%	0.53%	-0.062%	-0.051%	-0.045%	3.74%	12.08%	11.73%	0.532
Biennially	0.5	0.03%	0.80%	-0.085%	-0.060%	-0.047%	3.31%	12.39%	11.72%	0.557
Efficient (p=8)	3.31	0.02%	0.28%	-0.029%			4.01%	12.16%	11.67%	0.544
Efficient (p=4)	1.23	0.01%	0.52%		-0.022%		2.35%	12.08%	11.71%	0.535
Efficient (p=2)	0.42	0.00%	0.87%			-0.020%	0.87%	11.85%	11.47%	0.521
Buy & Hold	0	0	1.79%	-0.255%	-0.128%	-0.064%	0	12.26%	12.16%	0.520

identifies the rebalancing strategy. The next seven columns provide information for evaluating these strategies.

Columns two through four show the number of trades per year, annual trading costs (expressed as a percentage of portfolio value) and annualized tracking error. Trade-offs between trading costs and tracking error are apparent in each table. Among calendar strategies for a given target mix, trading costs fall and tracking error increases going from monthly to biennial rebalancing. Among efficient strategies for a given target mix, trades per year and trading costs fall while tracking error increases as the price of tracking accuracy declines from 8 to 2. This pattern among efficient strategies is expected. An efficiently rebalanced portfolio placing a higher value on tracking accuracy should trade more often, incur higher trading costs and have lower tracking error than an efficiently rebalanced portfolio with a lower price of tracking accuracy.

By construction, calendar strategies trade at regular calendar frequencies. Efficient strategies trade opportunistically, and beneficial rebalancing opportunities will not occur at uniform intervals. The irregular timing of trades will be a characteristic of efficient rebalancing strategies employing prices of tracking accuracy on the order of 2 to 8. For the medium equity ratio target mix, the efficient strategy with p=4 produced 39 trades during 1971-96. Vertical bars in figure 2 indicate the months when these trades occurred. Trades often were isolated, but sometimes trading occurred in bunches of two or more consecutive months (e.g., March-June 1983). At other times, long gaps existed between trades (e.g., October 1990-August 1994).

Figure 2



Comparing trading costs, the efficient strategies considered had lower annual trading costs than all corresponding calendar-based strategies.[5] In proportional terms, some of the differences in trading costs were large. (For example, the efficient strategies with p=8 had approximately the same number of trades per year as quarterly rebalancing, but the annual trading costs of these efficient strategies were only about 35 percent as large as those of quarterly rebalancing across the three target mixes.)

Comparing tracking errors, the results are mixed. The efficient strategies had higher annual tracking errors than monthly and quarterly rebalancing for each target mix. The efficient strategy with p=8 had equal or lower tracking errors than semiannual, annual and biennial rebalancing in all cases. The efficient strategy with p=4 had lower tracking errors than biennial rebalancing for all target mixes, and it had lower tracking error than annual rebalancing for the low equity ratio target. The efficient strategy with p=2 had *higher* annual tracking errors than all calendar strategies for each target mix.

When one rebalancing strategy has both lower trading costs and equal or lower tracking error than an alternative strategy, it *dominates* the alternative.[6] Efficient strategies with p=8 dominated the semiannual, annual and biennial strategies for all three target mixes. Efficient strategies with p=4 dominated the biennial strategy

[5]This finding is a result of the prices of tracking accuracy assumed. Efficient strategies with values of p sufficiently greater than *8* will have higher annual trading costs than some calendar strategies.

[6]Dominance also occurs when one rebalancing strategy has both lower tracking error and equal or lower trading costs than an alternative strategy.

for all three target mixes and the annual strategy for the low equity ratio mix. None of the efficient strategies with *p*=2 dominated a calendar strategy. No calendar-based strategy dominated an efficient strategy.

In the absence of dominance, a criterion is required to determine superiority in cases where one rebalancing strategy has lower trading costs and higher tracking error than an alternate strategy. *Realized utility* serves this function by providing a measure of the balance achieved between trading costs and tracking accuracy. Realized utility is computed by substituting annual trading costs and tracking error into equation (1).[7] Realized utility can be used to rank rebalancing strategies assuming that equation (1) correctly specifies the trade-off between trading costs and tracking accuracy.

Columns five through seven report *realized utility* for each of the three prices of tracking accuracy: *p*=8, 4 and 2. For a given price of tracking accuracy, the efficient strategy had higher realized utility than any competing calendar strategy for each of the three target mixes. Based on comparisons of realized utility, efficient rebalancing strategies could have provided superior combinations of trading costs and tracking accuracy than calendar-based strategies during 1971-96.

When the efficient strategies are ignored, realized utility comparisons indicate that annual rebalancing was superior to the other calendar strategies. For all three prices of tracking accuracy, annual rebalancing produced the highest realized utility for each of the three target mixes. (For *p*=8 and the low equity ratio mix, semiannual and annual rebalancing tied for the highest realized utility among the calendar strategies.) However, note that the buy-and-hold portfolio had higher realized utility than annual and other calendar rebalancing strategies for the high equity ratio mix when *p*=2 (table 4).

Annual turnover is reported in column eight. Among calendar strategies, turnover falls as the rebalancing frequency declines from monthly to biennially. Among efficient strategies, turnover falls as the price of tracking accuracy declines from 8 to 2. For all three target mixes, the efficient strategies with *p*=4 and *p*=2 had lower turnover than any calendar strategy. The efficient strategy with *p*=8 had lower turnover than monthly, quarterly and semiannual rebalancing for all target mixes, and it had lower turnover than annual rebalancing for the high equity ratio mix.

The monthly rebalancing strategies always had higher tracking error and turnover than the corresponding target mix portfolios (columns four and eight of tables 4-6).

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[7]For example, for the high equity ratio target mix (table 4), the realized utility for annual rebalancing when *p*=8 is computed as follows: $-.0006 - 8\,(.005)^2 = -.0008$ or –.08 percent. Since both trading costs and tracking error are detrimental, realized utility always will be negative. The closer realized utility is to zero, the better the balance between trading costs and tracking error.

Both the higher tracking error and turnover are due to trading costs. Every month, the monthly strategy's return is less than that of the target mix because the monthly strategy pays trading costs and the target mix does not. The differences in returns produce tracking error even though identical weights are applied to each asset class by the monthly strategy and the target mix every month. To provide funds for paying trading costs, the monthly strategy must sell a greater dollar value of assets than it buys every month. The monthly strategy's excess of sales over purchases results in higher turnover than the target mix (where sales always equal purchases).

The buy-and-hold portfolios had no rebalancing costs and larger tracking errors than all other portfolios. The tracking errors of the buy-and-hold portfolios range from 1.55 percent per year for the high equity ratio mix to 1.79 percent per year for the low equity ratio mix. None of the buy-and-hold portfolios produced higher realized utility than competing efficient rebalancing strategies, but buy-and-hold portfolios occasionally produced higher realized utility than one or more of the calendar strategies when $p=2$. In such cases, the cost savings of buy-and-hold more than made up for its greater tracking error.

The final three columns of tables 4-6 report standard performance summary measures: the annualized compound rate of return, the standard deviation of annual returns and the Sharpe ratio.[8] These performance measures are reported to satisfy many readers' interest in such information, but they should not be used to choose a rebalancing strategy for use in the future. This is because estimates of these statistics tend to be time-period specific, and what appear to be attractive average return-standard deviation combinations (e.g., those of biennial rebalancing in tables 4-6) cannot be relied upon to persist out of sample. Evidence reported below suggests that expected compound returns and standard deviations rise in rough proportion as the frequency of calendar rebalancing decreases or, for efficient strategies, as the price of tracking accuracy decreases. In addition, the efficient strategies have higher expected Sharpe ratios than buy-and-hold and all calendar strategies considered in this study.

Bootstrap Simulations

The 26-year period just examined is reasonably long, and, for many purposes, confidence could be placed in the findings from samples of such duration. But the historical simulations provide only one sample for each target mix, and a danger exists that some of the findings could be due to chance. To gain some sense of the reliability of particular relations observed among alternative rebalancing strategies, another type of simulation is employed: bootstrapping.

[8] One-month Treasury bill returns were used in computing Sharpe ratios.

Bootstrapping is a technique for using a limited amount of data to create additional samples so experiments can be repeated many times. The source data are the 1971-96 asset-class monthly returns. To construct a simulated history, each of the 312 months is assigned an equal probability of selection. One month (e.g., May 1983) is chosen at random, and the returns of each of the five asset-class portfolios for this month are recorded.[9] A second month (e.g., August 1977) is selected at random, and the asset-class returns for this month are recorded. This process is repeated until 240 months have been drawn so that a simulated 20-year history of returns is created. All drawings are done "with replacement" so that a given month can be chosen more than once in the 20-year sample. The rebalancing experiments with the various efficient and calendar strategies are repeated with the simulated 20-year sample. These operations are repeated until 100 trials have been conducted.

Bootstrapping has a number of desirable properties.

- Bootstrapping preserves the risk and return properties of the asset-class portfolios and the correlations among their returns.

- Potentially restrictive assumptions about the process generating returns (e.g., returns are drawings from normal distributions) are not required.

- Bootstrapping removes any spurious trends or reversals occurring during 1971-96 that may have affected the findings of the rebalancing simulations conducted with the actual history of returns.

Bootstrapping also removes any regularity (e.g., momentum or mean reversion) that is a true property of asset-class returns. The removal of mean reversion eliminates the potential for rebalancing to be a reliable source of "abnormal returns."

Tables 7-9 summarize the bootstrap simulations. These tables have the same format as tables 4-6, but the entries in tables 7-9 are averages of statistics produced in the 100 bootstrap trials. Averages of 100 trials should be more precise estimates of the expected values of the rebalancing parameters than the measures from the single-trial samples provided by the actual history of returns.

The second columns of tables 7-9 show the average number of trades per year for each rebalancing strategy. The efficient strategy with p=8 averaged about 3 trades per year, the efficient strategy with p=4 averaged about 1.3 trades per year, and the efficient strategy with p=2 averaged about one trade every two years. The

[9]The one-month Treasury bill return in the chosen month is recorded also for use in computing Sharpe ratios.

Table 7

High Equity Ratio Portfolios

Averages of 100 Bootstrap Simulations of 20-Year Holding Periods

Rebalancing Strategy	Trades Per Year	Annual Trading Costs	Tracking Error	Realized Utility			Turnover	Compound Return	Standard Deviation	Sharpe Ratio
				p=8	*p*=4	*p*=2				
Target Mix	12	0	0	0	0	0	12.15%	13.35%	13.03%	0.556
Monthly	12	0.13%	0.04%	-0.129%	-0.129%	-0.129%	12.21%	13.21%	13.01%	0.545
Quarterly	4	0.08%	0.17%	-0.080%	-0.079%	-0.079%	7.38%	13.27%	13.05%	0.550
Semiannually	2	0.06%	0.27%	-0.062%	-0.059%	-0.057%	5.32%	13.30%	13.09%	0.551
Annually	1	0.04%	0.40%	-0.053%	-0.047%	-0.043%	3.82%	13.32%	13.18%	0.552
Biennially	0.5	0.03%	0.58%	-0.056%	-0.042%	-0.035%	2.73%	13.35%	13.23%	0.551
Efficient (*p*=8)	3.02	0.03%	0.29%	-0.036%			3.57%	13.33%	13.07%	0.554
Efficient (*p*=4)	1.27	0.02%	0.55%		-0.028%		2.05%	13.36%	13.14%	0.555
Efficient (*p*=2)	0.52	0.01%	0.95%			-0.027%	1.16%	13.42%	13.28%	0.553
Buy & Hold	0	0	1.96%	-0.355%	-0.178%	-0.089%	0	13.70%	14.01%	0.544

Table 8

Medium Equity Ratio Portfolios

Averages of 100 Bootstrap Simulations of 20-Year Holding Periods

Rebalancing Strategy	Trades Per Year	Annual Trading Costs	Tracking Error	Realized Utility			Turnover	Compound Return	Standard Deviation	Sharpe Ratio
				p=8	*p*=4	*p*=2				
Target Mix	12	0	0	0	0	0	11.76%	12.70%	11.66%	0.549
Monthly	12	0.11%	0.04%	-0.109%	-0.109%	-0.109%	11.81%	12.58%	11.65%	0.539
Quarterly	4	0.07%	0.16%	-0.068%	-0.067%	-0.067%	7.17%	12.63%	11.69%	0.543
Semiannually	2	0.05%	0.25%	-0.053%	-0.050%	-0.049%	5.19%	12.66%	11.73%	0.544
Annually	1	0.03%	0.38%	-0.046%	-0.040%	-0.037%	3.72%	12.68%	11.83%	0.544
Biennially	0.5	0.02%	0.55%	-0.050%	-0.037%	-0.031%	2.68%	12.71%	11.89%	0.544
Efficient (*p*=8)	2.96	0.02%	0.28%	-0.030%			3.55%	12.69%	11.70%	0.547
Efficient (*p*=4)	1.33	0.01%	0.51%		-0.024%		2.13%	12.71%	11.76%	0.547
Efficient (*p*=2)	0.58	0.01%	0.90%			-0.024%	1.28%	12.75%	11.92%	0.545
Buy & Hold	0	0	2.02%	-0.372%	-0.186%	-0.093%	0	13.14%	12.86%	0.535

maximum and minimum trades per year in the 100 bootstrap trials for each efficient strategy reveal variation about these averages (table 10). This is another indication of the opportunistic character of efficient rebalancing. Over some extended holding periods, many profitable rebalancing opportunities will occur; over others, few will arise.

Table 9

Low Equity Ratio Portfolios

Averages of 100 Bootstrap Simulations of 20-Year Holding Periods

Rebalancing Strategy	Trades Per Year	Annual Trading Costs	Tracking Error	Realized Utility *p*=8	Realized Utility *p*=4	Realized Utility *p*=2	Turnover	Compound Return	Standard Deviation	Sharpe Ratio
Target Mix	12	0	0	0	0	0	11.03%	12.03%	10.45%	0.534
Monthly	12	0.09%	0.03%	-0.087%	-0.087%	-0.087%	11.08%	11.94%	10.44%	0.525
Quarterly	4	0.05%	0.15%	-0.055%	-0.054%	-0.053%	6.75%	11.98%	10.47%	0.528
Semiannually	2	0.04%	0.24%	-0.043%	-0.040%	-0.039%	4.89%	12.00%	10.51%	0.530
Annually	1	0.03%	0.36%	-0.038%	-0.033%	-0.030%	3.51%	12.01%	10.61%	0.529
Biennially	0.5	0.02%	0.52%	-0.043%	-0.031%	-0.025%	2.54%	12.04%	10.67%	0.528
Efficient (*p*=8)	2.71	0.02%	0.26%	-0.024%			3.40%	12.02%	10.48%	0.532
Efficient (*p*=4)	1.29	0.01%	0.49%		-0.021%		2.09%	12.03%	10.53%	0.532
Efficient (*p*=2)	0.58	0.01%	0.83%			-0.021%	1.31%	12.08%	10.67%	0.530
Buy & Hold	0	0	2.01%	-0.376%	-0.188%	-0.094%	0	12.51%	11.71%	0.520

Table 10

Maximum, Median and Minimum Trades Per Year in 100 Trials for Each Efficient Rebalancing Strategy

	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	*p*=4	*p*=2	*p*=8	*p*=4	*p*=2	*p*=8	*p*=4	*p*=2
Maximum	4.50	2.60	1.60	4.35	2.85	1.70	4.20	2.55	1.90
Median	3.10	1.20	0.45	2.92	1.25	0.45	2.70	1.20	0.45
Minimum	1.90	0.45	0.05	1.85	0.45	0.05	1.75	0.40	0.10

The third and fourth columns of tables 7-9 reveal trade-offs between trading costs and tracking error similar to those observed in tables 4-6. Among calendar strategies for the same target mix, average trading costs fall and average tracking error rises going from monthly to biennial rebalancing. Among efficient strategies for the same target mix, this inverse relation is apparent as the price of tracking accuracy decreases from 8 to 2.

The efficient strategies considered have lower average trading costs than competing calendar strategies. The efficient strategies with *p*=4 and *p*=2 have lower average trading costs than all calendar strategies. (The efficient strategy with *p*=4 averaged about 1.3 trades per year while the annual strategy always made one trade per year. Despite trading more frequently on average, the average ratio of this efficient strategy's trading costs to the annual strategy's trading costs was about .4

Table 11

Frequency in 100 Trials that Efficient Rebalancing Strategies had Lower Annual Trading Costs than Competing Calendar Strategies

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	100	100	100	100	100	100	100	100	100
Quarterly	100	100	100	100	100	100	100	100	100
Semiannually	100	100	100	100	100	100	100	100	100
Annually	100	100	100	100	100	100	100	100	100
Biennially	48	100	100	52	100	100	66	100	100

Table 12

Frequency in 100 Trials that Efficient Rebalancing Strategies had Lower Annual Tracking Error than Competing Calendar Strategies

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	0	0	0	0	0	0	0	0	0
Quarterly	0	0	0	1	0	0	1	0	0
Semiannually	32	0	0	27	0	0	26	0	0
Annually	94	2	0	97	5	0	95	7	0
Biennially	100	59	0	100	61	0	100	62	2

Table 13

Frequency in 100 Trials that Efficient Rebalancing Strategies "Dominated" Competing Calendar Strategies

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	0	0	0	0	0	0	0	0	0
Quarterly	0	0	0	1	0	0	1	0	0
Semiannually	32	0	0	27	0	0	26	0	0
Annually	94	2	0	97	5	0	95	7	0
Biennially	48	59	0	52	61	0	66	62	2

for each target mix.) The efficient strategy with p=8 has lower average trading costs than each calendar strategy except the biennial strategy where the averages are about the same.

The efficient strategies had lower annual trading costs than the calendar strategies in high proportions of the bootstrap experiments (table 11). The efficient strategies with p=4 and p=2 had lower trading costs than each calendar strategy in all 100 trials for all three target mixes. The efficient strategy with p=8 had lower trading costs than the monthly through annual strategies in all 100 trials for all three target mixes. Such high frequencies suggest probabilities near one that the trading cost advantages of the efficient strategies considered will persist in the future.

Referring to the fourth columns of tables 7-9, all efficient strategies considered had higher average tracking error than monthly, quarterly and semiannual rebalancing for each target mix, but some of the efficient strategies had lower average tracking error than annual or biennial rebalancing. The efficient strategy with p=8 had lower average tracking error than annual and biennial rebalancing for all three target mixes. This efficient strategy's tracking superiority over annual and biennial rebalancing occurred in at least 94 percent of the trials for each target mix (table 12).[10] The efficient strategy with p=4 also had lower average tracking error than biennial rebalancing for each target mix (tables 7-9), but it produced lower tracking errors than biennial rebalancing only in about 60 percent of the trials for each target (table 12). The efficient strategy with p=2 had *higher* average tracking error than all calendar strategies for each target (table 7-9), and the probability that it will have lower tracking error than any of the calendar strategies considered appears to be near zero (table 12).

Recall that a rebalancing strategy having both lower tracking error and equal or lower trading costs than an alternate strategy "dominates" the alternate strategy. In terms of the averages reported in tables 7-9, the efficient strategy with p=8 appears to dominate annual and biennial rebalancing for all three targets, and the efficient strategy with p=4 appears to dominate biennial rebalancing for all three targets. But only the dominance of annual rebalancing by the efficient strategy with p=8 appears to have a high probability (table 13). The efficient strategy with p=8 dominated annual rebalancing in at least 94 percent of the trials for each target mix. In the other instances of apparent dominance, the probabilities may not differ much from 50 percent. (The efficient strategy with p=8 dominated biennial rebalancing in about one-half to two-thirds of the trials, and the efficient strategy with p=4 dominated

[10]In the historical simulations, the efficient strategy with p=8 had equal or lower tracking errors than semiannual rebalancing for all three target mixes (tables 4-6), but that result appears to be due to chance. The bootstrap simulations indicate that the average tracking errors of the efficient strategy with p=8 are greater than those of semiannual rebalancing (tables 7-9). They also suggest that this efficient strategy can have lower tracking error than semiannual rebalancing about 25 to 30 percent of the time (table 12).

Table 14

Frequency in 100 Trials that Efficient Rebalancing Strategies had Higher Realized Utility than Competing Calendar Strategies and Buy and Hold

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	100	100	100	100	100	100	100	100	100
Quarterly	100	100	100	100	100	100	100	100	100
Semiannually	100	100	100	100	100	100	100	100	100
Annually	100	100	98	100	100	91	100	100	98
Biennially	100	99	82	100	100	83	100	99	82
Buy & Hold	100	100	100	100	100	100	100	100	100

Table 15

Frequency in 100 Trials that Efficient Rebalancing Strategies had Lower Annual Turnover than Competing Calendar Strategies

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	100	100	100	100	100	100	100	100	100
Quarterly	100	100	100	100	100	100	100	100	100
Semiannually	100	100	100	100	100	100	100	100	100
Annually	72	100	100	64	100	100	64	100	100
Biennially	1	95	100	0	93	100	1	88	100

Table 16

Frequency in 100 Trials that Each Calendar Strategy or Buy and Hold had the Highest Realized Utility when Efficient Rebalancing Strategies are Ignored

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	0	0	0	0	0	0	0	0	0
Quarterly	0	0	0	0	0	0	0	0	0
Semiannually	5	0	0	6	1	0	8	2	0
Annually	55	23	3	57	23	3	60	29	5
Biennially	40	73	79	37	75	84	32	69	84
Buy & Hold	0	4	18	0	1	13	0	0	11

biennial rebalancing in about 60 percent of the trials.) The efficient strategy with p=2 rarely dominated a calendar strategy, and no calendar strategy ever dominated one of the efficient strategies considered in this study.

The efficient rebalancing strategies considered can be expected to deliver higher realized utility than calendar and buy-and-hold strategies. In columns five through seven of tables 7-9, the average realized utility of each efficient strategy is greater than the average realized utility of any competing calendar or buy-and-hold strategy. The probabilities that the efficient strategies will produce higher realized utility than the calendar and buy-and-hold strategies appear to be high (table 14). In terms of turnover, the efficient strategies considered can be expected to be superior to most calendar strategies. Referring to column eight of tables 7-9, the efficient strategies with p=4 and p=2 had lower average turnover than all calendar strategies, and the frequencies that these two efficient strategies had lower turnover than each calendar strategy were high (table 15). The average turnover of the efficient strategy with p=8 was lower than all calendar strategies except biennial rebalancing. The probabilities that this efficient strategy will have lower turnover than monthly, quarterly and semiannual rebalancing appear to be near one, and the probabilities that it will have lower turnover than biennial rebalancing appear to be near zero.

When the efficient strategies are excluded from consideration, the bootstrap simulations suggest that either annual or biennial rebalancing can be expected to yield the highest realized utility (columns four through six of tables 7-9). The price of tracking accuracy is important in determining which is the best of the calendar strategies.[11] When p=8, annual rebalancing had the highest average realized utility for all three target mixes, and annual rebalancing yielded the highest realized utility in between 55 and 60 percent of the trials for each target mix (table 16). Biennial rebalancing yielded the highest realized utility in most other cases when p=8. When p=4 or 2, biennial rebalancing had the highest average realized utility for all three target mixes. When p=4, biennial rebalancing yielded the highest realized utility in about 70 percent of the trials, and annual rebalancing yielded the highest realized utility in most other cases. When p=2, biennial rebalancing yielded the highest realized utility in about 80 percent of the trials, and the buy-and-hold strategy yielded the highest realized utility in most other cases. Semiannual rebalancing rarely yielded the highest realized utility, and monthly and quarterly rebalancing never produced the highest realized utility.

[11]This finding differs from that of the earlier historical simulations (tables 4-6). There, annual rebalancing produced the highest realized utility for all three prices of tracking accuracy for all three target mixes. Here, biennial rebalancing also can be expected to yield the highest realized utility at prices of tracking accuracy of 4 or less.

Table 17

Frequency in 100 Trials that Buy and Hold had Higher Realized Utility than Each Competing Calendar Strategy

Rebalancing Strategy	High Equity Ratio Target			Medium Equity Ratio Target			Low Equity Ratio Target		
	p=8	p=4	p=2	p=8	p=4	p=2	p=8	p=4	p=2
Monthly	17	52	79	13	35	71	10	30	61
Quarterly	6	22	62	2	19	48	0	15	37
Semiannually	3	13	46	0	10	30	0	8	25
Annually	0	8	26	0	4	21	0	2	18
Biennially	0	4	18	0	1	13	0	0	11

Table 18

Maximum, Median and Minimum Annual Tracking Error in 100 Trials for Buy and Hold

	High Equity Ratio Target	Medium Equity Ratio Target	Low Equity Ratio Target
Maximum	4.51%	4.64%	4.57%
Median	1.80%	1.83%	1.90%
Minimum	0.79%	0.88%	0.87%

The lower the price of tracking accuracy, the greater the probability that buy and hold will yield higher realized utility than some calendar strategies (table 17). When p=2, buy and hold had higher realized utility than monthly rebalancing in 79 percent of the trials for the high equity ratio target and 71 percent for the medium target. When an investor assigns a sufficiently low value to tracking accuracy, too much rebalancing can be worse than no rebalancing.

The average tracking errors of the buy-and-hold strategies were about 2 percent per year (column three of tables 7-9). The tracking errors of buy and hold ranged between 4.6 percent per year and 85 basis points per year (table 18).

The last three columns of tables 7-9 report averages of performance statistics. Among calendar strategies for a given target mix, average compound returns and standard deviations increase approximately in proportion as the frequency of rebalancing decreases from monthly through biennially. For each target mix, the average Sharpe ratios of the calendar strategies are equal to the second decimal place. Among efficient strategies for a given target mix, average compound returns and standard deviations increase approximately in proportion as the price of tracking accuracy declines from 8 to 2. For each target mix, the average Sharpe ratios for the three efficient strategies are equal to the second decimal place. Although the differences are small, the average Sharpe ratios of the efficient strategies exceed those of competing calendar strategies and buy and hold for each target mix.[12] These findings suggest that large differences in historical performance statistics (such as those reported in tables 4-6) are due to chance and are unreliable criteria for choosing a rebalancing strategy.

Conclusions

Many investors should benefit by switching to an efficient rebalancing strategy from a conventional calendar-based approach. Efficient rebalancing strategies are expected to yield better balances between trading costs and tracking accuracy over long holding periods than calendar rules. The superiority of efficient strategies in terms of realized utility was demonstrated in historical rebalancing simulations for the 1971-96 period. In bootstrap simulations, some efficient strategies (with prices of tracking accuracy of 8 and 4) had higher realized utility than each calendar strategy in at least 99 out of 100 trials.

Efficient strategies are more cost-conscious than most calendar strategies. Some efficient strategies (with p=4 and 2) had lower annual trading costs than each calendar strategy in 100 percent of the bootstrap trials. Although averaging more trades per year, an efficient strategy (with p=4) had annual trading costs that were only about 40 percent as large as those of annual rebalancing on average. In part, the trading cost differences are due to the fact that efficient strategies make partial adjustments to target weights while calendar strategies make full adjustments.

Among calendar strategies exclusively, less frequent rebalancing schedules are superior to more frequent schedules. For the prices of tracking accuracy considered (p=8, 4 and 2), annual or biennial rebalancing will yield the highest realized utility in most cases.

The consistency of findings across the three target mixes considered suggests that they may generalize to a variety of target mixes. Nonetheless, the selection of an efficient rebalancing strategy should be based on experiments with specific target mixes and appropriate trading costs.

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[12] Comparing the efficient strategies with p=8 and 4 with calendar strategies and buy and hold, all but one of the differences in average Sharpe ratios are statistically significant at the .05 level. The exception occurs with the efficient strategy with p=4 and semiannual rebalancing for the low equity ratio target.

Appendix

Rebalancing should occur whenever its benefits outweigh its costs. Rebalancing is beneficial because it enables the actual portfolio to "track" more closely the returns of the target mix. Tracking accuracy is measured by *predicted tracking variance* (V):

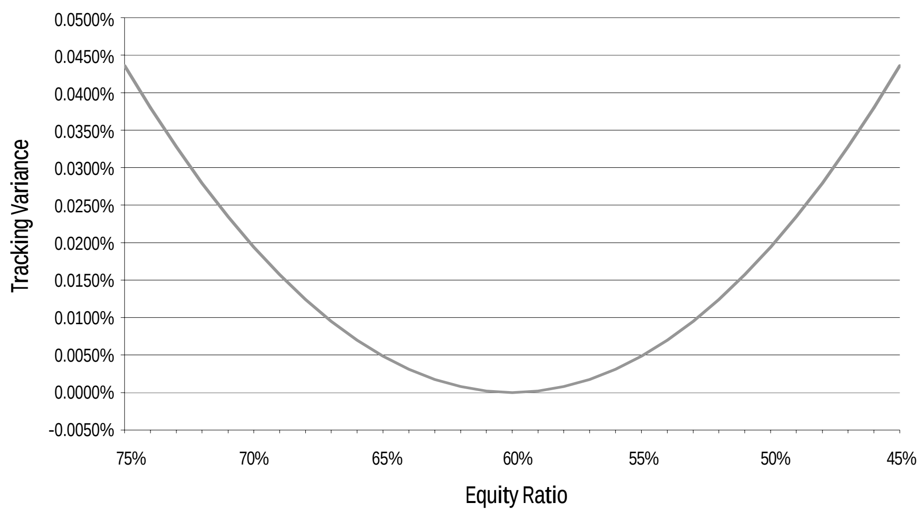
$$(2)\quad V = \sum_i \sum_j (w_i - w_i^*)(w_j - w_j^*) Cov_{i,j}.$$

In (2), w_i is the actual weight of asset i, w_i^* is the target weight of asset i, and $Cov_{i,j}$ is the predicted covariance between the annual returns of assets i and j.

Tracking variance is a *u*-shaped function of the differences between actual and target weights (figure 3). Tracking variance approaches zero as the target weights are neared, but the function flattens out in the vicinity of the zero. This indicates that the incremental gains in tracking accuracy become smaller and smaller as the target weights are approached. With proportional trading costs, the final minute gains in tracking accuracy become prohibitively expensive.

Figure 3

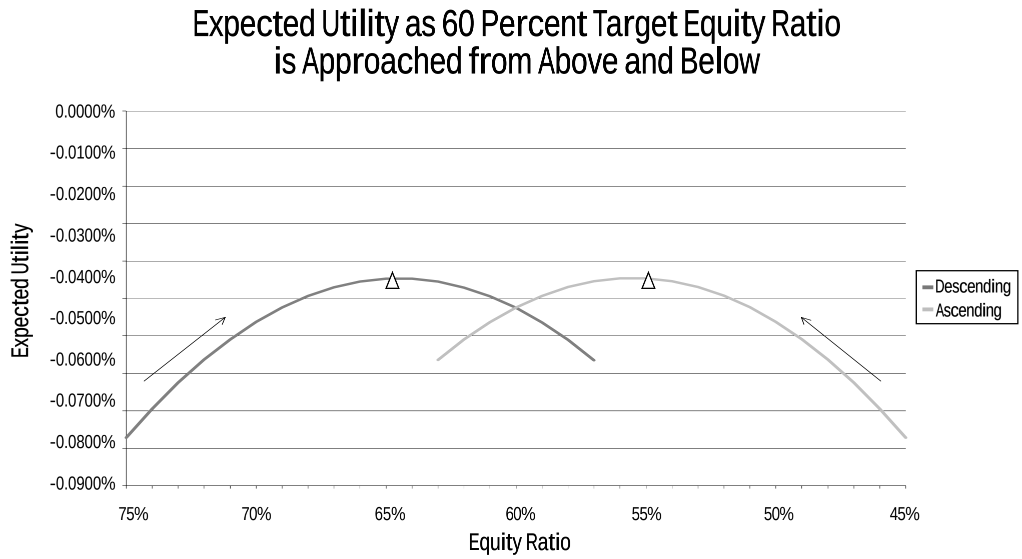
Tracking Variance in the Vicinity of a 60% Target Equity Ratio



Rebalancing costs are the total costs of trading including commissions, fees and market impact. *Trading costs* (C) are assumed to be proportional to the dollar value of assets traded, and they are expressed as fractions of the market value of the portfolio.

It is assumed that an investor's assessment of the tradeoffs between the benefits and costs of rebalancing can be expressed by a *utility function* (U):

Figure 4



(3) $U = -C - pV.$

In (3), p is the investor's *price of tracking accuracy*. The investor's objective in rebalancing decisions is to maximize the value of U.

At any time, determining if rebalancing is beneficial is a multi-step process. First, the utility of the current portfolio without rebalancing (U_0) is calculated. Since holding the current portfolio involves no trading, U_0 depends only on the current portfolio's tracking variance (V_0):

(4) $U_0 = -pV_0.$

Second, the maximum utility possible from rebalancing (U_R) is determined.[13] U_R depends on the rebalancing costs required (C_R) and the tracking variance of the rebalanced portfolio (V_R):

(4) $U_R = -C_R - pV_R.$

Finally, U_0 and U_R are compared. If U_0 is greater than or equal to U_R, rebalancing is not worthwhile, and no adjustments should be made to the current portfolio. Rebalancing should occur only when U_R exceeds U_0, and the trades required to attain U_R should be executed.

The rebalancing trades that maximize utility make partial adjustments to target weights (figure 4). A target portfolio of 60 percent equities and 40 percent bonds

[13]Maximum utility with rebalancing is attained when the incremental increase in rebalancing costs equals the value of the incremental reduction in tracking variance. That is, $dU = 0$ when $dC = -p \cdot dV$.

is assumed. If the current equity ratio is 75 percent, utility can be increased by reducing the equity ratio to 65 percent. If the equity ratio is reduced below 65 percent, utility falls. The decline in utility is due to the flattening of the tracking variance function in the vicinity of the 60 percent target equity ratio (figure 3). Likewise, beginning at a 45 percent equity ratio, utility can be increased by raising the equity ratio to 55 percent, but increasing the equity ratio beyond 55 percent causes utility to decline (figure 4). The 65 percent and 55 percent equity ratios where utility is maximized are the limits of the non-trading region (NTR) about the 60 percent target equity ratio illustrated in figure 1 above.

The width of the NTR's bands depends on trading costs and the investor's price of tracking accuracy. Everything else constant, the bands will be narrower if:

- trading costs are reduced or

- the price of tracking accuracy increases.

Narrower bands increase the frequency of advantageous rebalancing opportunities.

Related Papers

Constantinides, G. "Multiperiod Consumption and Investment Behavior with Convex Transactions Costs." *Management Science* 25 (1979), 1127-37.

Davis, M. and A. Norman. "Portfolio Selection with Transaction Costs." *Mathematics of Operations Research* 15 (1990), 676-713.

Leland, H. "Option Pricing and Replication with Transaction Costs." *Journal of Finance* 40 (1985), 1283-1302.

Leland, H. "Multiple Asset Rebalancing in the Presence of Transactions Costs and Capital Gains Taxes." Haas School of Business Working Paper. University of California, Berkeley. September 1997.

McCalla, D. "Enhancing the Efficient Frontier with Portfolio Rebalancing." *Journal of Pension Plan Investing* 1, No. 4 (1997), 16-32.

Sources and Descriptions of Data

S&P 500 Index

Courtesy of Roger G. Ibbotson and Rex A. Sinquefield, Stocks, Bonds, Bills, and Inflation: The Past and the Future, Dow Jones, 1989. Ibbotson Associates, Chicago, annually updates work by Roger G. Ibbotson and Rex A. Sinquefield. Used with permission. All rights reserved.

Large Cap Value Subtrust

July 1963-March 1992: Fama-French Large Cap Value Strategy. Simulates Dimensional's hold range and estimated trading costs. Courtesy of Fama-French and CRSP: Deciles 1-5 Size, (.7) BtM.
April 1992-Present: Dimensional's Large Cap Value Trust.

U.S. 9-10 Small Company Portfolio

Small Company Universe (Deciles 9 & 10)- all exchanges.
July 1962- December 1972: CRSP Database, NYSE & AMEX, rebalanced quarterly.
January 1973-December 1981: CRSP Database, NYSE & AMEX & OTC, rebalanced quarterly.
January 1982-Present: U.S. 9-10 Small Company Portfolio net of all fees.

EAFE Index

Courtesy of Morgan Stanley Capital International. Europe, Australia, and Far East Index net dividends ($).

Lehman Government/Corporation Bond Index*

Average maturity: 8-13 years. Courtesy of Lehman Brothers, Inc.

Thirty-Day Treasury Bills

Courtesy of Roger G. Ibbotson and Rex A. Sinquefield, Stocks, Bonds, Bills, and Inflation: The Past and the Future, Dow Jones, 1989. Ibbotson Associates, Chicago, annually updates work by Roger G. Ibbotson and Rex A. Sinquefield. Used with permission. All rights reserved.

Intermediate Term Government Bond Returns

Courtesy of Roger G. Ibbotson and Rex A. Sinquefield, Stocks, Bonds, Bills, and Inflation: The Past and the Future, Dow Jones, 1989. Ibbotson Associates, Chicago, annually updates work by Roger G. Ibbotson and Rex A. Sinquefield. Used with permission. All rights reserved.

Long Term Corporate Bond Returns:

Average mautrity: 20 years.
Courtesy of Roger G. Ibbotson and Rex A. Sinquefield, Stocks, Bonds, Bills, and Inflation: The Past and the Future, Dow Jones, 1989. Ibbotson Associates, Chicago, annually updates work by Roger G. Ibbotson and Rex A. Sinquefield. Used with permission. All rights reserved.

*For 1970-72, a weighted average of the total returns of government and corporate bonds was used as a proxy for the Lehman index. The proxy was composed of 70 percent intermediate-term government bonds and 30 percent long-term corporate bonds rebalanced monthly.