Please refer to the prospectus for information on Dimensional Funds, including investment policies, charges, expenses, risks and other matters of interest to the prospective investor. This material is provided solely as background information for registered investment advisors, institutional investors and other sophisticated investors and is not intended for public use. It should not be distributed to investors of products managed by Dimensional Fund Advisors Inc. or to potential investors. © 2001 by Dimensional Fund Advisors Inc. All rights reserved. Unauthorized copying, reproducing, duplicating or transmitting of this material is prohibited.

SEPTEMBER 2001

Rebalancing: When, How & Why – Part 1

TRUMAN A. CLARK

MANY FINANCIAL ADVISORS RECOMMEND PORTFOLIOS combining asset classes in fixed proportions, e.g., 60 percent domestic equities, 10 percent international equities and 30 percent bonds. Once such a portfolio is formed, asset proportions will diverge from the target proportions as security prices fluctuate. Left free to drift, a portfolio can evolve into an asset mix with decidedly different risk and return characteristics than the target mix.

Rebalancing refers to the adjustment of a portfolio's asset proportions toward the target proportions. In the absence of costs, a portfolio could be rebalanced continuously and never deviate from its target weights. But rebalancing does entail costs so continuous rebalancing is not feasible.[1] Financial advisors must determine *when* and *how* to rebalance. A rational approach to rebalancing decisions involves weighing benefits and costs.

When to Rebalance

Rebalancing must be beneficial because investors and financial advisors do it.[2] Its costs include some or all of the following: trading costs (i.e., load fees, commissions, etc.), administrative fees, the value of time and other resources expended in processing rebalancing orders, and capital-gains taxes. Once the benefits and costs have been identified, it is rational to rebalance only when benefits exceed costs.

---

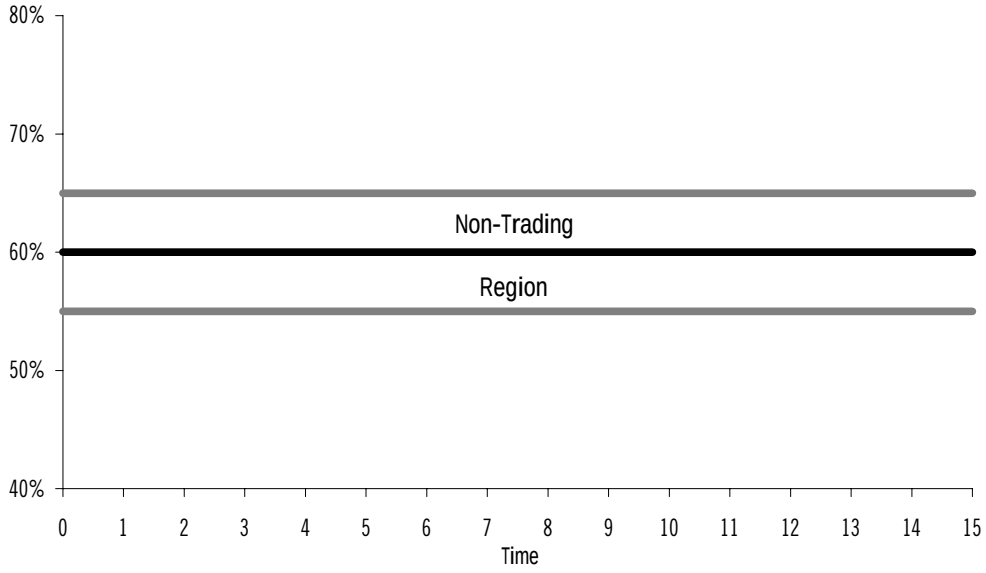
[1] Leland (1985) demonstrates that an infinite amount of trading is required to keep assets continuously at their target proportions. The fact that only a mad man would attempt to rebalance continuously is *prima facie* evidence that rebalancing is costly.

[2] Part 3 of this paper will discuss the benefits of rebalancing.

Costs create a *non-trading region* (or *NTR*) around the target weights. To illustrate, a target portfolio of 60 percent equities and 40 bonds is assumed (figure 1). The NTR boundaries about the target equity ratio are 65 percent and 55 percent. At each boundary, the costs of the smallest possible rebalancing trade exactly equal the benefits it produces—rebalancing is a break-even proposition. For equity ratios within the bands, the costs of rebalancing exceed the benefits so no trading occurs. As long as the portfolio's equity ratio remains between 65 percent and 55 percent, it is allowed to meander about the 60 percent target ratio. It is only when the equity ratio rises above 65 percent or falls below 55 percent that benefits exceed costs and rebalancing is profitable.

Figure 1

Non-Trading Region of a Stock-Bond Portfolio with a 60-Percent Target Equity Ratio



The occasions and frequency of profitable rebalancing opportunities cannot be predicted reliably. Their occurrence depends on random relative movements of asset-class values. When asset-class returns diverge quickly and repeatedly, profitable rebalancing opportunities may occur frequently—perhaps several times within a month or quarter. When asset-class returns tend to move together over prolonged periods, profitable rebalancing opportunities may occur infrequently—perhaps once every two or more years.

Rebalancing at pre-determined calendar intervals is not a rational approach. At times, it will produce excessive trading when costs exceed benefits. At other times,

it will not trade enough and miss profitable rebalancing opportunities. Financial advisors should not abdicate their responsibilities to make rational decisions on behalf of their clients by adopting mechanical, calendar-based rebalancing rules.

How to Rebalance

Given a profitable rebalancing opportunity, it must be decided whether to restore each asset to its precise target weight (*full adjustment*) or to move some or all assets part way toward their targets (*partial adjustment*). The nature of rebalancing costs determines whether full or partial adjustments should be made.

Fixed costs – Costs that are constant dollar amounts independent of the value of assets traded are *fixed*. For example, if each set of rebalancing transactions costs a flat $50, the costs remain $50 if $1 or $100 million worth of assets are exchanged. If benefits exceed $50, all trading required to restore the target weights will be made to maximize the surplus of benefits over costs. **When costs are fixed, it pays to make full adjustment to the target weights whenever benefits exceed costs.**

In figure 1, if costs are fixed, the equity ratio is returned to 60 percent whenever it wanders above 65 percent or below 55 percent. At the target ratio, rebalancing benefits *net of costs* are maximized. But full adjustment is warranted only when all costs are fixed.

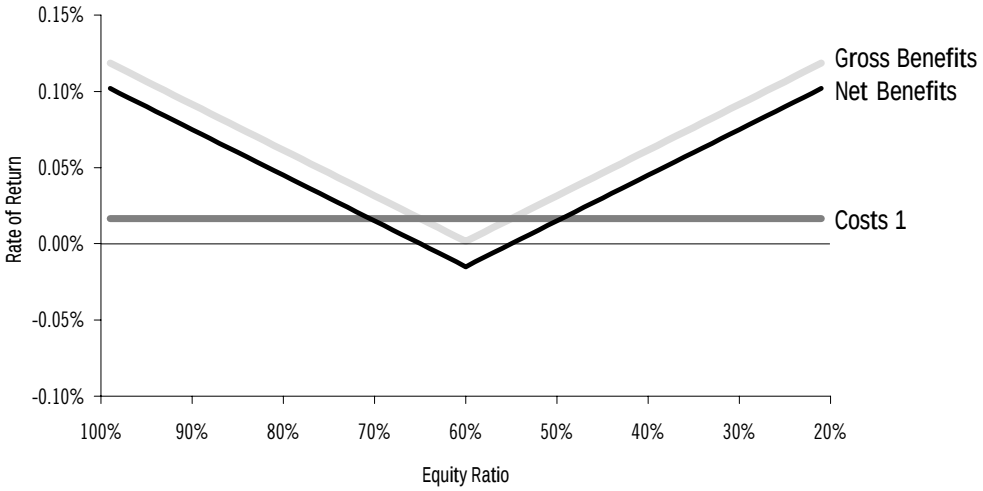
Proportional costs – Costs that increase with the value of assets traded are *proportional*. For example, if each trade costs 50 basis points of the value of assets exchanged, it costs $5 to trade $1,000 worth of assets and $500 to trade $100,000 worth. **When costs are proportional, it pays to make only partial adjustment toward the target proportions whenever benefits exceed costs**. To see why this is so, the interaction of proportional costs and benefits must be understood.

In the two-asset case with a 60-percent target equity ratio, rebalancing yields positive *gross benefits* as a portfolio's equity ratio is moved toward the target ratio (figure 2). Far from the target ratio (*e.g.*, 95 percent or 25 percent), gross benefits are large for each initial percentage point change of the equity ratio. As the target ratio is neared from above or below, the gross benefits from each additional percentage point change of the equity ratio become progressively smaller and approach zero.

Proportional costs are assumed. The costs of each percentage point change of the equity ratio are constant and trace a horizontal line as the equity ratio is increased or reduced (figure 2).

Figure 2

The Benefits & Costs of Rebalancing a Stock-Bond Portfolio with a 60% Target Equity Ratio



The *net benefits* of rebalancing equal gross benefits minus costs. It pays to change the equity ratio to the point where net benefits equal zero. As the equity ratio is reduced from 95 percent, net benefits equal zero when the equity ratio reaches 65 percent. Reducing the equity ratio further yields negative net benefits because costs exceed gross benefits. Full adjustment to the 60-percent target equity ratio is not worth the additional expense.

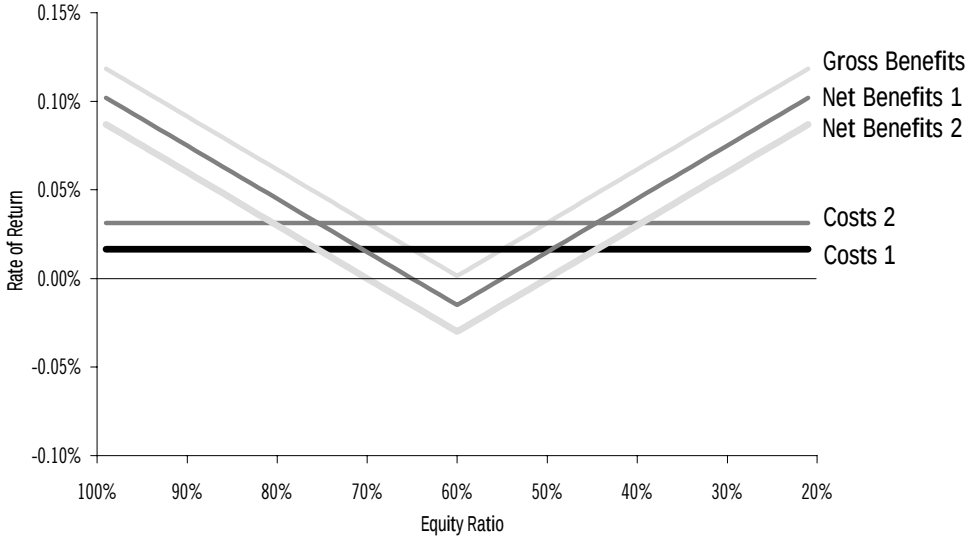
The results are similar if the target ratio is approached from below. Beginning at a 25-percent equity ratio, net benefits equal zero when the equity ratio reaches 55 percent. Further increases of the equity ratio yield negative net benefits. It does not pay to make full adjustment to the 60-percent target equity ratio.

The 65-percent and 55-percent equity ratios where net benefits are zero in figure 2 establish the boundaries of the non-trading region illustrated in figure 1. The width of the NTR is determined by the interaction of benefits and costs. When costs increase to Costs 2 from Costs 1, Net Benefits 2 equal zero at equity ratios of 70 percent and 50 percent (figure 3). These equity ratios are the boundaries of the NTR given the increased costs. **For a given schedule of gross benefits, greater costs widen the NTR. For a given schedule of costs, reduced gross benefits widen the NTR**.[3] Wider bands reduce the frequency of profitable rebalancing opportunities.

[3] In part 3 of this paper, it will be shown that benefits are affected by an investor's attitude toward risk. The more aggressive the investor is, the less the benefits of rebalancing are and the wider the NTR will be.

Figure 3

The Benefits & Costs of Rebalancing a Stock-Bond Portfolio with a 60% Target Equity Ratio: The Effects of Higher Costs



In figure 1, equity ratios greater than 65 percent or less than 55 percent represent profitable rebalancing opportunities. With proportional costs, the equity ratio should be brought to the nearer boundary of the NTR and moved no further.[4]

Some financial advisors routinely make full adjustments whenever client accounts are rebalanced. They do not recognize that full adjustments are justified only when rebalancing costs are fixed. Full adjustments are not in their clients' interests with proportional costs.

Identifying Fixed and Proportional Rebalancing Costs

Many clients of financial advisors implement their asset-allocation strategies entirely with mutual funds. For these investors, most elements of rebalancing costs are relatively easy to identify as fixed or proportional and to measure.

---

[4] For target portfolios composed of more than two assets, the non-trading region is more complicated than the simple band illustrated in figure 1. With more than two assets, a multi-dimensional non-trading surface surrounds the target ratios. The interaction of benefits and costs determines the size and shape of this surface. Whenever the gross benefits of rebalancing exceed its costs, asset weights should be brought back to this surface.

Trading costs – Trading costs due to commissions and market impact are virtually zero for most mutual-fund investors.[5] Trading costs are incurred if load or reimbursement fees are levied on purchases or redemption charges are assessed on sales. Such fees usually are proportional to the value of assets traded.

Trading costs may differ from fund to fund within a mutual funds' family, e.g., load fees may be charged for some funds and not others. The costs of buying and selling the same mutual fund need not be the same, e.g., loads are charged for purchases but not sales.

Reimbursement fees deserve attention. All of Dimensional's mutual funds are no-load and do not charge for redemptions. But proportional reimbursement fees are assessed on purchases of some of Dimensional's international funds. Reimbursement fees are levied on all purchases of these funds including those connected with exchanges of assets between funds by established clients. These fees currently range from 50 to 100 basis points of a fund's offering price. Buyers of funds with reimbursement fees face proportional costs that are not faced by sellers of the same funds. For all of Dimensional's other mutual funds, trading costs are zero.

Administrative costs – Often these are zero, but some custodians charge for recording changes in mutual fund holdings. Such charges tend to be levied in small, fixed-dollar amounts (e.g., $30 per transaction).

Rebalancing order management costs – A financial advisor expends some time and other resources in placing and recording rebalancing orders on behalf of a client. With modern communications and low-cost, high-speed computers, these costs should be low and fixed.

Capital-Gains Taxes – Rebalancing entails the sale of "winners" and the purchase of "losers."[6] The sale of winners often results in the realization of capital gains. Capital-gains realizations increase with the value of assets sold, and they may increase at accelerating rates as more and more assets are sold. In taxable accounts, capital gains are subject to federal tax rates of 20 percent or more, and they may be subject to additional state or local taxation. **Capital-gains taxes are proportional costs, and they can produce wide non-trading regions around the target ratios of some asset classes**.

---

[5] Investors in a mutual fund share *pro rata* in reductions in NAV caused by trading. Unless an investor owns a large portion of a mutual fund's shares, he may assume rightly that his portion of the trading costs generated by his decision to trade is zero.

[6] A "winner" is an asset with a portfolio weight exceeding its target weight. A "loser" is an asset with a weight less than its target.

Please refer to the prospectus for information on Dimensional Funds, including investment policies, charges, expenses, risks and other matters of interest to the prospective investor. This material is provided solely as background information for registered investment advisors, institutional investors and other sophisticated investors and is not intended for public use. It should not be distributed to investors of products managed by Dimensional Fund Advisors Inc. or to potential investors. © 2001 by Dimensional Fund Advisors Inc. All rights reserved. Unauthorized copying, reproducing, duplicating or transmitting of this material is prohibited.

SEPTEMBER 2001

Rebalancing: When, How & Why – Part 2

TRUMAN A. CLARK

Rebalancing Tax-Exempt Accounts

FOR PORTFOLIOS COMPOSED ENTIRELY OF NO-LOAD, NO-REDEMPTION fee mutual funds in the same funds' family, rebalancing costs are low. With low costs, profitable rebalancing opportunities are apt to arise frequently.

Administrative fees (if any) and rebalancing order management expenses may be the only components of costs. These are fixed costs in most cases, and full adjustment to target weights should be made whenever gross benefits exceed costs.

Reimbursement fees create an exception. If rebalancing trades involve purchase of a fund charging a proportional reimbursement fee, only a partial adjustment is warranted.

Rebalancing Taxable Accounts

Capital-gains taxes can increase greatly the costs of rebalancing taxable accounts. They also introduce a number of complexities into the rebalancing decision-making process. But some rebalancing transactions do not result in increased capital-gains liabilities.

Tax-free rebalancing – Contributions, trading in the aftermath of a steep market decline and the availability of previously realized tax losses create opportunities for rebalancing without an increase in capital-gains tax liabilities. Without capital-gains taxes, rebalancing costs for taxable and tax-exempt accounts should be the same. These costs are low and mostly fixed.

Contributions result only in purchases so no capital gains are realized. Portfolio balance can be improved by buying losers in greater proportions than their current portfolio weights. If a contribution is motivated solely to rebalance a portfolio, all costs associated with the resulting purchases are attributable to rebalancing. If a contribution is motivated by a decision to increase the size of the portfolio, only the incremental costs (if any) incurred in rebalancing are relevant to the rebalancing decision. For example, a client is following a plan that calls for $100,000 annual contributions. The costs of distributing the cash inflow proportionately across existing holdings are $100. If the funds are distributed to improve balance by buying relatively more losers than winners, the costs are $150. The incremental costs attributable to rebalancing are $50, and $50 should be weighed against the benefits of rebalancing.

A deep, widespread decline in the market values of most asset classes may create opportunities for rebalancing without realizing large capital gains. When most asset classes lose value, winners are the asset classes that decline less than losers. If the current prices of such winners are slightly above or at their cost bases, their sales produce little or no gains. If their current prices are less than their cost bases, the sales of winners produce capital losses. Since realized losses can be used to offset future gains, they are valuable, and the realization of losses reduces the costs (or increases the benefits) of rebalancing. In such extreme circumstances, full adjustment may be warranted to garner the benefits of being at the targets while booking large losses.

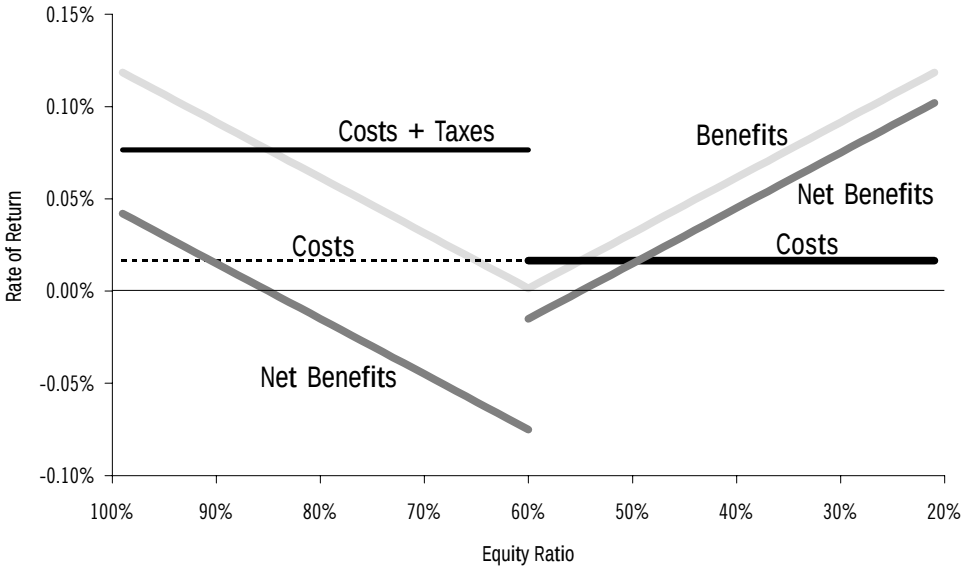
When an account has a stockpile of realized capital losses, rebalancing can be conducted without increasing capital-gains tax liabilities. Previous losses can be used to offset the gains realized by the sale of winners. But the use of realized losses in rebalancing may entail opportunity costs. For example, a client has planned future redemptions that will result in the realization of capital gains. Inventoried losses would be available to offset these gains if the losses were not used in prior rebalancing. The opportunity costs of using losses in rebalancing should not be overlooked.

Rebalancing with Taxes – If tax-free opportunities do not exist, rebalancing produces capital-gains tax liabilities that increase with the value of assets sold. Given tax rates of 20 percent or higher, capital-gains taxes can make it much costlier to rebalance a taxable account than a tax-exempt account. As a result, the non-trading region of a taxable account will be wider than that of a comparable tax-exempt account. This can be illustrated in the case of two assets considered earlier.

A portfolio is composed of stocks and bonds, and the target equity ratio is 60 percent. Starting above the target ratio, the costs of each percentage point reduction of the equity ratio are equal (figure 4). In addition, each percentage point reduction of the equity ratio requires the sale of equities producing capital gains. Because of taxes on these gains, each percentage point reduction of the equity ratio reduces returns by 6 basis points. Starting at 95 percent, costs plus taxes equal gross benefits when the equity ratio reaches 85 percent, and this is the upper boundary of the NTR. The 85-percent upper boundary with capital-gains taxes is higher than the 65-percent upper boundary without taxes (figure 2). The determination of the lower boundary will be discussed in the next section.

Figure 4

The Benefits & Costs of Rebalancing a Stock-Bond Portfolio with a 60% Target Equity Ratio and Capital-Gains Taxes on Equity Sales



The uniqueness of taxable accounts - Capital-gains taxes complicate the management of taxable accounts. Each account will have a unique schedule of rebalancing costs resulting in a unique NTR. Each account will require individual monitoring to identify its beneficial rebalancing opportunities.

For each client, the tax consequences of selling differ across asset classes. For example, over long periods, equity sales are likely to produce greater capital gains per dollar sold than bond sales. As a result, the boundaries of a client's NTR will not be symmetric about the target ratios of all asset classes. To illustrate, assume that bonds can be sold without realizing capital gains (figure 4). Beginning at 25

percent, each percentage point increase of the equity ratio requires constant costs, but no tax liabilities are incurred. Net benefits equal zero at a 55-percent equity ratio. Because the tax consequences of selling differ for stocks and bonds, the boundaries of the NTR are 85 percent and 55 percent. Compared to a tax-exempt account with 65-percent and 55-percent boundaries (figure 2), capital-gains taxes create a wider, asymmetric NTR.

As time passes, because of contributions, reinvestment of distributions and rebalancing, the holdings of each asset class in a portfolio become composed of tiers with different cost bases (Table 1). To limit capital gains, deliveries are made from the highest basis tranche ($23,750) for initial sales up to $25,000.[7] For sales from $25,001 to $50,000, deliveries are made from the second highest stratum ($22,500). As sales increase further, deliveries are made from layers with successively lower bases, and realized capital gains and taxes get larger and larger.

Table 1

Capital-Gains Taxes Resulting from Increasing Sales from Equity Holdings with a Mixture of Cost Bases

Market Value	Cost Basis	Capital Gains	Cumulative Taxes*	Taxes per Dollar Sold
$25,000	$23,750	$1,250	$250	1.00%
$25,000	$22,500	$2,500	$750	1.50%
$25,000	$20,000	$5,000	$1,750	2.33%
$100,000	$60,000	$40,000	$9,750	5.57%
$250,000	$100,000	$150,000	$39,750	9.35%
$575,000	$143,750	$431,250	$126,000	12.60%

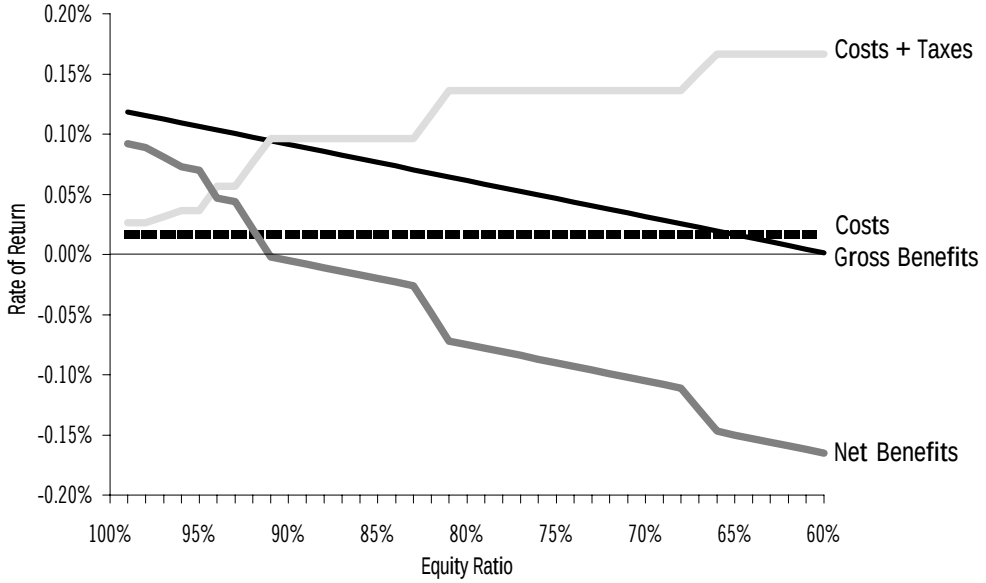
*The capital-gains tax rate is 20 percent.

In the stock-bond case with multiple cost bases, the taxes required to achieve successive percentage point reductions of the equity ratio increase in steps at irregular intervals (figure 5). Assuming that an investor with the stock holdings in Table 1 wants to rebalance from 100-percent equities toward a 60-percent target equity ratio, costs plus increasing taxes equal gross benefits at 91 percent. Further reductions of the equity ratio yield negative net benefits because of prohibitively high and increasing taxes.

[7] It is assumed that all capital gains are long-term.

Figure 5

The Benefits & Costs of Rebalancing a Stock-Bond Portfolio
Given Increasing Capital-Gains Taxes on Equity Sales



For a given asset class, small rebalancing sales that do not exhaust the highest cost-basis tiers result in little or no taxes. Larger rebalancing sales that require tapping lower cost-basis tranches result in rapidly increasing taxes. In a taxable account, small rebalancing trades may yield positive net benefits, but large rebalancing trades may be too costly to be justified.

The age of a taxable account is likely to affect rebalancing decisions. In its early years, a portfolio may be composed of core holdings that have not appreciated greatly so large rebalancing trades can be made with limited capital gains. As the account matures, the gap between the market value and cost basis of its core will tend to widen so only small rebalancing sales can be made without realizing large gains. Under these circumstances, older taxable accounts will have wider NTRs than younger taxable accounts. Younger accounts will be rebalanced more fully than mature accounts.

Periodic contributions to taxable accounts are doubly useful for rebalancing. As discussed earlier, contributions initially enable rebalancing without the realization of capital gains. In addition, each contribution creates a new layer of assets with cost bases at current market values. In periods of rising asset values, these higher basis tiers permit rebalancing sales that produce smaller capital gains. Over a range of sales, deliveries from higher basis tranches created by contributions can

keep the tax consequences of rebalancing relatively low as a taxable account matures. Different contribution patterns create schedules of rebalancing costs plus taxes that differ across accounts of the same age. If for no other reason than to reduce the present and future costs of rebalancing, financial advisors should encourage clients to make frequent contributions to taxable accounts.

Each taxable account is unique due to its asset-class composition, availability of tax losses and particular cost-basis structure. The tax liabilities generated by rebalancing differ from account to account so each account has its own peculiar NTR. No simple rebalancing rule (e.g., fully rebalance all taxable accounts annually) can be in the interests of all clients. Time must be devoted to monitoring the rebalancing opportunities of each taxable account. A financial advisor's fees should reflect the higher costs of properly managing taxable accounts.

Redemptions – Redemptions may be used to improve portfolio balance, but redemptions often result in the realization of capital gains. If relatively more winners than losers are sold to move the portfolio toward the target mix, more capital gains will be realized. The increase in taxes is an incremental cost of rebalancing. For example, a planned redemption of $100,000 will result in $12,000 of capital-gains taxes if a slice of the existing portfolio is sold. If relatively more winners are sold than losers, capital-gains taxes increase to $14,000. The incremental cost of rebalancing is $2,000.

Coordinated Management of Tax-Exempt and Taxable Accounts

A financial advisor can be of great assistance to a client having both tax-exempt and taxable accounts. Benefits accrue through the coordinated management of accounts as a single portfolio rather than as separate, unrelated portfolios. The judicious allocation of asset classes with relatively high expected taxable distributions to tax-exempt accounts and asset classes with relatively low expected taxable distributions to taxable accounts can increase the expected after-tax rate of return of the composite portfolio. Coordinated management of the two classes of accounts also will permit rebalancing of the aggregate portfolio with reduced capital-gains realizations.

To illustrate, suppose a client has a $200,000 tax-exempt account and an $800,000 taxable account (Table 2).[8] If the accounts are viewed as separate entities, the initial distribution of asset classes in each account might be strictly proportional (Table 2, Panel A). But interest constitutes large proportions of the returns of

[8] The various distributions of assets in Table 2 are not based on careful analysis. The numbers are chosen to illustrate some ideas. Readers are encouraged to evaluate the allocations critically and to develop reasoned alternatives.

Table 2

Hypothetical Initial Asset Allocation Between Paired Tax-Exempt and Taxable Accounts (Amounts in Thousands)

A. Strictly Proportional

Tax-Exempt		Taxable		Combined	Target	Difference
Enhanced Large Market	$60	Large Market	$240	$300	$300	$0
Large Value	$20	TM MW Value	$80	$100	$100	$0
Small Market	$40	TM Small Market	$160	$200	$200	$0
Small Value	$20	TM Small Value	$80	$100	$100	$0
Micro-Cap	$20	Micro-Cap	$80	$100	$100	$0
Fixed Income	$40	Fixed Income	$160	$200	$200	$0
Account Total	$200	Account Total	$800	$1,000	$1,000	$0

B. Modified for Tax Considerations

Tax-Exempt		Taxable		Combined	Target	Difference
Enhanced Large Market	$0	Large Market	$300	$300	$300	$0
Large Value	$50	TM MW Value	$50	$100	$100	$0
Small Market	$50	TM Small Market	$150	$200	$200	$0
Small Value	$0	TM Small Value	$100	$100	$100	$0
Micro-Cap	$0	Micro-Cap	$100	$100	$100	$0
Fixed Income	$100	Fixed Income	$100	$200	$200	$0
Account Total	$200	Account Total	$800	$1,000	$1,000	$0

C. Modified for Tax and Future Rebalancing Considerations

Tax-Exempt		Taxable		Combined	Target	Difference
Enhanced Large Market	$40	Large Market	$235	$275	$300	-$25
Large Value	$20	TM MW Value	$75	$95	$100	-$5
Small Market	$30	TM Small Market	$160	$190	$200	-$10
Small Value	$20	TM Small Value	$75	$95	$100	-$5
Micro-Cap	$20	Micro-Cap	$75	$95	$100	-$5
Fixed Income	$70	Fixed Income	$180	$250	$200	$50
Account Total	$200	Account Total	$800	$1,000	$1,000	$0

short-term bond funds, and interest is taxed at income-tax rates. Assuming that large-cap value and small-cap market funds have relatively high dividend and capital distributions per dollar of market value, their returns streams are exposed to considerable income and capital-gains taxation, too. An initial distribution of assets placing relatively more fixed income, large value and small market in the tax-exempt account might increase the after-tax rate of return of the composite portfolio (Table 2, Panel B).

Planning for future rebalancing requirements also may influence the initial distribution of assets. As discussed earlier, rebalancing trades are less costly in the tax-exempt account than in the taxable account. To the fullest extent possible, rebalancing trades for the aggregate portfolio should be conducted in the tax-exempt account. For effective rebalancing, the tax-exempt account must hold asset classes that are highly correlated with asset classes in the taxable account.

The tax-exempt account often is much smaller than the taxable account, and adjustments in the tax-exempt account alone may not be large enough to restore desirable balance in the aggregate portfolio. In such cases, rebalancing needs may compete with taxes in determining the initial distribution of assets. For future rebalancing purposes, perhaps the tax-exempt account will hold somewhat less fixed income, large value and small market and somewhat more of other equity asset classes than an exclusive focus on taxes suggests (Table 2, Panel C). If regular contributions to the taxable account are not envisioned, it even may be advantageous initially to over-weight fixed income and to under-weight equities in the composite portfolio. Short-term bond funds often can be sold with little or no capital gains. The sale of surplus fixed-income holdings in the taxable account releases funds to buy equity losers and improve aggregate portfolio balance at low cost. It is conceivable that the increase in rebalancing flexibility warrants the additional taxes incurred by holding initially a larger proportion of fixed-income funds in the taxable account.[9] At times, the tax sting of holding an excess of fixed income in the taxable account can be mitigated with municipal bond funds.

Coordinated management of a client's tax-exempt and taxable accounts can increase the expected after-tax rate of return and reduce the rebalancing costs of the client's aggregate portfolio. Such potential improvements are limited when taxable accounts are much larger than tax-exempt accounts. But even if the gains are small, a conscientious financial advisor will make sure that they are exploited fully.

---

[9] Leland (1999) provides rigorous proof that initial over-weighting of cash and under-weighting of risky assets may be desirable for future rebalancing purposes given capital-gains taxes and trading costs that differ across asset classes.

Please refer to the prospectus for information on Dimensional Funds, including investment policies, charges, expenses, risks and other matters of interest to the prospective investor. This material is provided solely as background information for registered investment advisors, institutional investors and other sophisticated investors and is not intended for public use. It should not be distributed to investors of products managed by Dimensional Fund Advisors Inc. or to potential investors. © 2001 by Dimensional Fund Advisors Inc. All rights reserved. Unauthorized copying, reproducing, duplicating or transmitting of this material is prohibited.

SEPTEMBER 2001

Rebalancing: When, How & Why – Part 3

TRUMAN A. CLARK

The Benefits of Rebalancing

The costs of rebalancing are objective and measurable. Its benefits are subjective and difficult to quantify. Despite these obstacles, the benefits of rebalancing must be probed to answer the question *why* a portfolio should be rebalanced from time to time.

Some financial advisors say that rebalancing is beneficial because it allows their clients "to sleep better at night." This simple expression contains important insights. It suggests that the benefits of rebalancing derive from controlling risk. It also suggests that a client's attitude toward risk and reward influences the value placed on rebalancing.

To flesh out these intuitive concepts, assumptions must be made concerning how to measure risk and how to represent an individual's tolerance for risk. Examples follow showing how schedules of benefits can be derived from two frequently used measures of risk. Readers are warned that these examples are *ad hoc*. They are intended only to illustrate how one might go from a measure of risk to an operational definition of rebalancing benefits. ***These examples are not recommended for implementation!***

Portfolio Variance as a Measure of Risk

Many financial advisors think of risk and reward in terms of portfolio variance and expected return. For a stock-bond portfolio, portfolio expected return (E) is a function of the equity ratio (e) and the expected returns of stocks and bonds (Table 3, Column 2). Portfolio variance (V) is a function of the equity ratio and the covariance matrix of stock and bond returns (Table 3, Column 3).

Table 3

Gross Benefits of Rebalancing Stock-Bond Portfolios for Mean-Variance Investors with Different Prices of Risk

Equity Ratio	Expected Return	Variance	Aggressive p=1.182 Benefit	Cautious p=1.923 Benefit
75%	0.128	0.0393		
74%	0.127	0.0385	0.00005	
73%	0.126	0.0377	0.00003	
72%	0.125	0.0369	0.00002	
71%	0.124	0.0362	0.00001	
70%	0.123	0.0354	0.00000	
69%	0.122	0.0346	0.00002	
68%	0.121	0.0339	0.00003	
67%	0.120	0.0332	0.00005	
66%	0.119	0.0325	0.00006	
65%	0.119	0.0317		
45%	0.101	0.0195		
44%	0.100	0.0190		0.00008
43%	0.099	0.0185		0.00006
42%	0.098	0.0180		0.00004
41%	0.097	0.0176		0.00002
40%	0.096	0.0171		0.00000
39%	0.095	0.0166		0.00002
38%	0.094	0.0162		0.00004
37%	0.093	0.0158		0.00006
36%	0.092	0.0153		0.00008
35%	0.092	0.0149		

Asset Class	Expected Return	Variance	Covariance
Stocks	0.15	0.0625	
Bonds	0.06	0.0064	0.01

Portfolio Expected Return = e x E(Stocks) + (1-e) x E(Bonds).

Portfolio Variance = e^2 x V(Stocks) + $(1-e)^2$ x V(Bonds) + 2 x e x (1-e) x Cov(Stocks, Bonds).

Benefit = $E_1 - E_0 - p \times (V_1 - V_0)$.

It is assumed that an individual's attitude toward risk and return is expressed by a constant *price of risk*. The price of risk (p) indicates how much expected return an investor will pay to reduce variance by one unit. A timid investor is extremely sensitive to risk so his price of risk is relatively high. An aggressive investor is less sensitive to risk so his price is relatively low.

Moving toward the target portfolio, the gross benefit of rebalancing to portfolio 1 from portfolio 0 is the change in expected return minus the value of the change in variance:

(1) Benefit = $E_1 - E_0 - p$ x $(V_1 - V_0)$.

For a given price of risk, rebalancing benefits approach zero in the vicinity of the target portfolio. For a more aggressive investor with a price of risk of 1.182, the target portfolio has a 70-percent equity ratio (Table 3, Column 4).[10] A more cautious investor has a higher price of risk (1.923), and the target equity ratio is 40 percent (Table 3, Column 5). As the price of risk increases, the target equity ratio decreases.

In the section "How to Rebalance" in the part one of this paper, reasons for the existence of a non-trading region (NTR) were discussed. It was noted that the width of the NTR is determined by the interaction of benefits and costs. For a given schedule of gross benefits, greater costs widen the NTR. Now it will be shown that an investor's attitude toward risk affects the gross benefits of rebalancing and the width of the NTR.

Figure 6 displays benefits schedules for two prices of risk and constant rebalancing costs for each percentage point change in the equity ratio. For a more aggressive investor with a low (1.182) price of risk, rebalancing benefits equal costs at 85-percent and 55-percent equity ratios—15 percentage points above and below the 70-percent target ratio. For a more cautious investor with a higher price of risk (1.923) and 40-percent target, benefits equal costs at 50 percent and 30 percent—10 percentage points away from the target.

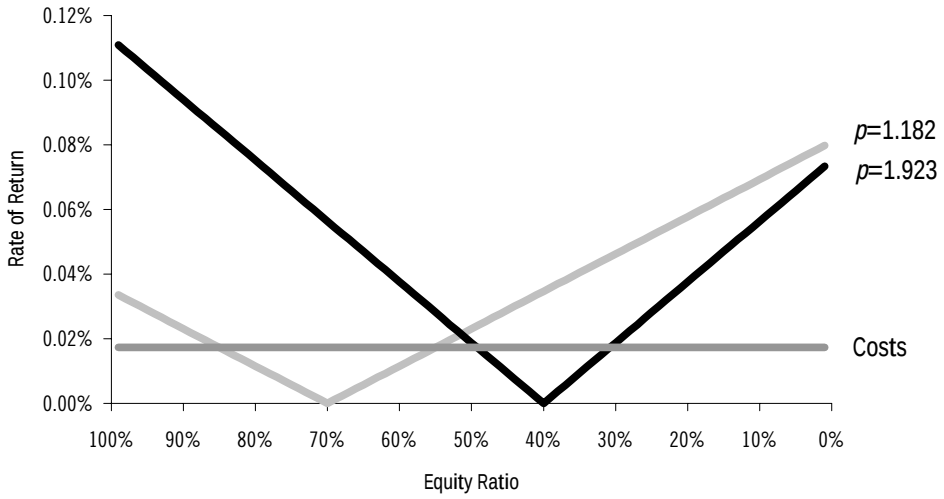
An aggressive investor does not place a high value on risk control and has a wide NTR. A cautious investor values risk control more highly and has a narrower NTR. Holding everything else the same, **the more aggressive the investor is, the wider the NTR will be.**

---

[10] At the target portfolio, the ratio of the change in expected return to the change in variance equals the price of risk: *i.e.*, $dE/dV = p$. A 1.182 price of risk is chosen to obtain a 70-percent target equity ratio given the assumed expected returns, variances and covariance (Table 3). The other price of risk (1.923) is chosen to obtain a 40-percent target equity ratio.

Figure 6

The Benefits & Costs of Rebalancing Stock-Bond Portfolios for Investors with Different Prices of Risk



Tracking Variance as a Measure of Risk

Institutional investors and their consultants often view risk in terms of tracking accuracy. A target portfolio (or benchmark) is adopted, and *tracking error* is the difference between the return of an investor's portfolio and the return of the benchmark over a given time period. *Tracking standard deviation* is the standard deviation of the time-series of tracking error. Tracking standard deviation measures how closely the returns of a portfolio mimic or "track" the returns of the target portfolio.[11]

For stock-bond portfolios, tracking standard deviation (T) is a function of the target equity ratio (e^*), the equity ratio of another portfolio (e) and the covariance matrix of stock and bond returns. Assuming a 60-percent target equity ratio, the computation of tracking standard deviation is illustrated in Table 4, Column 2. Tracking standard deviation is zero at the target equity ratio, and it increases symmetrically as the equity ratio moves above or below the target.

---

[11] A portfolio tracks a target portfolio perfectly when every return of the portfolio equals the return of the target plus a constant (or *alpha*). In such cases, tracking standard deviation is zero. With a sample of monthly returns, tracking standard deviation (T) can be estimated as:

$$T = \sqrt{\frac{12}{N-1}\sum_{t=1}^{N}\left(R_{P,t} - R_t^* - \overline{R}_P + \overline{R}^*\right)^2}\ .$$

$R_{P,t}$ is the return of portfolio P in month t, R_t^* is the return of the target portfolio in month t, $\overline{R}_P$ and $\overline{R}^*$ are the sample means of the returns of portfolio P and the target, N is the number of months in the sample period, and the sum of squares is multiplied by 12 to "annualize" the estimate.

Table 4

Gross Benefits of Rebalancing a Stock-Bond Portfolio with a 60-Percent Target Equity Ratio for Investors with Different Prices of Tracking Accuracy

Equity Ratio	Tracking Std. Dev.	Tracking Variance	Aggressive t=1.0 Benefit	Cautious t=2.0 Benefit
65%	0.01106	0.00012		
64%	0.00885	0.00008	0.00004	0.00009
63%	0.00663	0.00004	0.00003	0.00007
62%	0.00442	0.00002	0.00002	0.00005
61%	0.00221	0.00000	0.00001	0.00003
60%	0	0	0.00000	0.00001
59%	0.00221	0.00000	0.00001	0.00003
58%	0.00442	0.00002	0.00002	0.00005
57%	0.00663	0.00004	0.00003	0.00007
56%	0.00885	0.00008	0.00004	0.00009
55%	0.01106	0.00012		

Asset Class	Variance	Covariance
Stocks	0.0625	
Bonds	0.0064	0.01

Target equity ratio (e^*) = 60%.
Tracking Variance = $(e\text{-}e^*)^2$ x [V(Stocks) + V(Bonds) - 2 x Cov(Stocks, Bonds)].
Tracking Standard Deviation (T) = Square Root (Tracking Variance).
Benefit = - t x (T_1 - T_0).

Moving in the direction of the target portfolio, the gross benefit of rebalancing to portfolio 1 from portfolio 0 is the value of the reduction in tracking standard deviation squared (or *tracking variance*).

(2) Benefit = - t x (T_1^2 - T_0^2).

It is assumed that an investor's attitude toward tracking risk is summarized by a constant *price of tracking accuracy* (t).[12] This price indicates how much value an investor places on a one-unit reduction in tracking variance. For a more aggressive investor, the price of tracking accuracy (1.0) is relatively low. For a more cautious investor, the price of tracking accuracy (2.0) is higher. Assuming a 60-percent target equity ratio, rebalancing benefits for the two prices of tracking accuracy are shown in Table 4, Columns 4 and 5.

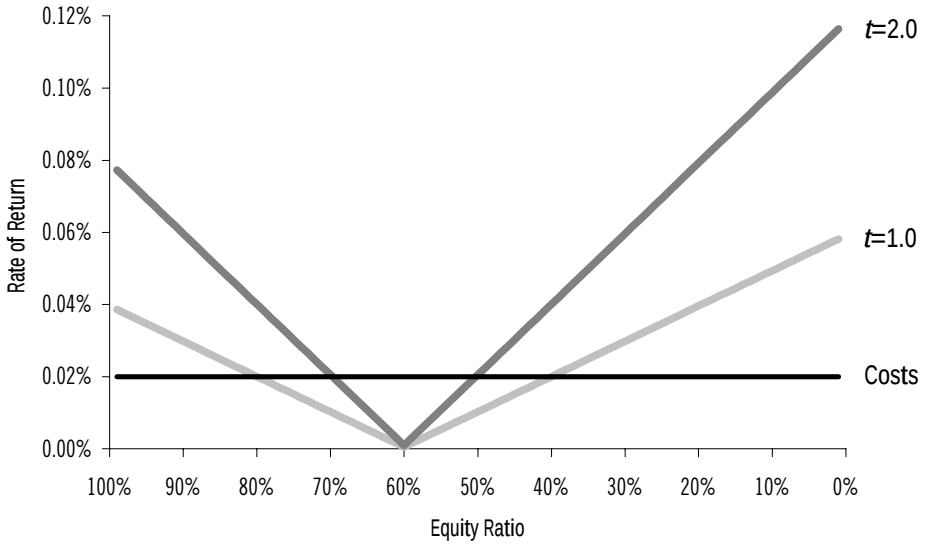
[12] Some financial experts contend that investors may have different attitudes toward total risk (or variance) and tracking risk. This is controversial, but, to allow for this possibility, different symbols are used for the prices of risk and tracking accuracy in equations (1) and (2).

Figure 7 displays the gross benefits of rebalancing for the two prices of tracking accuracy. Rebalancing costs are constant for each percentage point change in the equity ratio. An investor's attitude toward risk again influences the width of the NTR. An aggressive investor does not place a high value on tracking accuracy and has a wide NTR (80 percent to 40 percent). A cautious investor values tracking accuracy more highly and has a narrower NTR (70 percent to 50 percent). Holding everything else the same, the more aggressive the investor is, the wider the NTR will be.

Investors' attitudes toward risk affect the benefits of rebalancing. Each client may have a unique taste for risk requiring unique rebalancing decisions for his tax-exempt and taxable accounts.[13] **Universal rebalancing rules applied to all tax-exempt or taxable accounts cannot be in the interests of all clients.**

Figure 7

The Benefits & Costs of Rebalancing Stock-Bond Portfolio with a 60% Target Equity Ratio for Investors with Different Prices of Tracking Accuracy



Pitfalls in the Use of Portfolio Variance and Tracking Variance

Financial advisors considering the use of either a mean-variance or tracking-variance framework to solve rebalancing problems should proceed with caution. As

[13] In the section "The Uniqueness of Taxable Accounts" in part two of this paper, it was argued that taxes cause rebalancing costs to differ across taxable accounts. Differences in attitudes toward risk cause rebalancing decisions to differ from client to client for both tax-exempt and taxable accounts.

with solutions from "optimizers" in portfolio selection, the reliability and practicality of rebalancing decisions made by either approach depend critically on the accuracy of statistical inputs. Relatively small errors in the estimation of expected returns, standard deviations or covariances may produce large errors in recommended rebalancing trades.[14] Even if input statistics can be estimated with some precision today, these parameters may not be stable over time. Time-varying parameters may cause erroneous rebalancing choices. **Estimation error and non-stationarity are formidable obstacles to the successful application of mean-variance or tracking-variance techniques to the solution of rebalancing problems.**

A more fundamental problem exists. The findings of Fama and French (1992) imply that variance is not a complete measure of risk. The behavior of stock returns is explained well by three risk factors (*i.e.*, the excess return of the market over Treasury bills, the excess return of small-capitalization stocks over large-capitalization stocks and the excess return of high book-to-market stocks over low book-to-market stocks). An asset's risk is measured by its sensitivity to each of these factors. Because neither provides a complete measure of risk, **neither the mean-variance nor the tracking-variance approach can yield a complete or accurate specification of the benefits of rebalancing.**

It might be possible to apply the Fama-French three-factor model to rebalancing problems. A quadratic loss function could be used estimate rebalancing benefits. For each risk factor, a target sensitivity and price for departures from that target sensitivity would be specified. The loss function would penalize squared departures from the target sensitivities, and rebalancing benefits would be measured by reductions in loss. This is a speculative suggestion. Estimation errors and unstable parameters still would be serious obstacles to successful implementation.

Rebalancing and Compound Returns

Rebalancing is a risk-control instrument, and improved risk control can increase a portfolio's compound (or realized) rate of return. The compound return is important because it links today's investment to future wealth. An asset's or a portfolio's compound return is related to its expected return approximately as:

(3) Compound Return = Expected Return – 0.5 x Variance.[15]

---

[14] Optimizer results are notoriously sensitive to trivial changes in expected returns. Optimizer results tend to be less sensitive to similar changes in standard deviations and covariances. In the tracking-variance approach, the calculation of rebalancing benefits in equation (2) does not require expected returns. If standard deviations and covariances can be estimated more accurately than expected returns, the tracking-variance approach may be superior to the mean-variance approach in some rebalancing applications.

[15] This approximation provides good estimates with returns simulated by drawings from normal distributions. With actual returns, it tends to provide better estimates for short-period (daily or monthly) returns than for long-period (quarterly or annual) returns.

Booth and Fama (1992) define the *diversification return* as the difference between the compound return of a portfolio and a weighted average of the compound returns of its components. Assume that temporary departures from a portfolio's target weights increase variance without affecting the expected rate of return. Such an increase in variance can be viewed as the consequence of tracking error. Rebalancing will boost a portfolio's compound return and its diversification return if it reduces tracking variance.

Consider a stock-bond portfolio with a 60-percent target equity ratio (Table 5). The target portfolio's expected rate of return is 11.40 percent per year. (With zero tracking standard deviation, the compound return is 10.38 percent, and the diversification return is 99 basis points). With tracking standard deviation of 5 percent per year, the compound return is about 10.26 percent, and the diversification return is 86 basis points. If rebalancing reduces tracking standard deviation to 2 percent from 5 percent, the compound return increases to about 10.36 percent from 10.26 percent, and the diversification return increases to 97 basis points from 86 basis points. Over a 25-year time span, this 10-basis point increase in the compound return translates into an additional 28 cents (or $11.77 minus $11.49) in the terminal value of each dollar invested initially.

Table 5

Compound Returns and the Diversification Benefits of Rebalancing

Target Expected Return	Target Variance	Tracking Std. Dev.	Tracking Variance	Combined Variance	Portfolio Compound Return	Average Compound Return	Diversification Return	Growth of $1 Over 25 Years
0.114	0.0203	0.000	0.0000	0.0203	0.1038	0.0940	0.0099	$11.82
0.114	0.0203	0.010	0.0001	0.0204	0.1038	0.0940	0.0098	$11.81
0.114	0.0203	0.020	0.0004	0.0207	0.1036	0.0940	0.0097	$11.77
0.114	0.0203	0.030	0.0009	0.0212	0.1034	0.0940	0.0094	$11.70
0.114	0.0203	0.040	0.0016	0.0219	0.1030	0.0940	0.0091	$11.61
0.114	0.0203	0.050	0.0025	0.0228	0.1026	0.0940	0.0086	$11.49
0.114	0.0203	0.075	0.0056	0.0259	0.1010	0.0940	0.0071	$11.09
0.114	0.0203	0.100	0.0100	0.0303	0.0988	0.0940	0.0049	$10.55

Asset Class	Expected Return	Variance	Covariance	Compound Return
Stocks	0.15	0.0625		0.11875
Bonds	0.06	0.0064	0.01	0.0568

Target Equity Ratio (e^*) = 0.6.
Target Expected Return = e^* x E(Stocks) + (1-e^*) x E(Bonds).
Target Variance = e^{*2} x V(Stocks) + (1-e^*)^2 x V(Bonds) + 2 x e^* x (1-e^*) x Cov(Stocks,Bonds).
Combined Variance = Target Variance + Tracking Variance.
Compound Return = Expected Return - 0.5 * Variance.
Portfolio Compound Return = Target Expected Return - 0.5 x Combined Variance.
Average Compound Return = e^* x Compound Return (Stocks) + (1-e^*) x Compound Return (Bonds).
Diversification Return = Portfolio Compound Return - Average Compound Return.

But the costs of rebalancing must be considered. Suppose that without rebalancing, tracking standard deviation is 5 percent, and the compound return is 10.26 percent (Table 5). Assume that each percentage-point reduction in tracking standard deviation requires additional rebalancing costs that reduce returns by 3 basis points per year.[16] Reducing tracking standard deviation to 2 percent from 5 percent costs 9 basis points. This is a worthwhile expenditure because the *net* compound return increases to 10.27 percent (or 10.36 percent minus .09 percent) from 10.26 percent. It would not make sense to reduce tracking standard deviation further to one percent from 2 percent. This reduction in tracking standard deviation would cost an additional 3 basis points per year and reduce the net compound return to 10.26 percent (or 10.38 percent minus .12 percent) from 10.27 percent.

The examples in Table 5 indicate that incremental increases in compound return diminish as tracking standard deviation approaches zero: measured in single basis points rather than in tens of basis points. Unless rebalancing costs are correspondingly small, rebalancing's ability to increase *net* compound returns is limited. For taxable accounts where rebalancing costs often are high, improper rebalancing will reduce net compound returns.

These examples are useful for pedagogic purposes, but they are based on simplifying, unrealistic assumptions. Departures from target weights increase or reduce variance and increase or reduce the expected rate of return. Rebalancing serves to dampen these changes, but shifting expected returns and variances complicate isolation and measurement of the effects of rebalancing on compound return. Despite these complications, one should be skeptical of claims that rebalancing can increase net compound returns by 100 basis points per year or more.

Rebalancing and Expected Returns

The proposition that a rebalancing strategy can increase expected return is dubious. One thing is certain: rebalancing entails costs, and costs reduce expected rates of return.

Despite its costs, proponents of the proposition that rebalancing increases expected returns point to evidence of long-term mean reversion in asset returns. By selling winners to buy losers, rebalancing increases holdings of losers with temporarily increased expected returns while lightening up on holdings of winners with temporarily reduced expected returns. Over long time periods, rebalancing tends to increase expected returns by exploiting the effects of mean reversion.

---

[16] Rebalancing costs reduce the expected rate of return and its associated compound return by equal amounts.

Some mean-version advocates go further. They argue that it pays to over-weight losers and to under-weight winners relative to their target weights. It is doubtful that mean reversion is sufficiently reliable and strong to warrant such aggressive rebalancing or "tactical asset allocation" strategies. The evidence of long-term mean reversion also must be weighed against the evidence of short-term momentum in asset returns. Improperly timed rebalancing may result in getting out of winners before they complete their runs and into losers that continue to yield disappointing returns. All of this smacks of market timing: an appealing strategy with historical data and on paper but one that is devilishly difficult to bring off successfully in real time with real money on the line.

Rebalancing costs definitely reduce expected returns. Rebalancing also may produce offsetting increases in expected returns by capturing long-term mean reversion. The net effects of rebalancing on expected returns are uncertain, but **it seems unlikely that rebalancing strategies can produce big increases in net expected returns.**

Simulations of Rebalancing Strategies

Simulations with historical returns may provide misleading estimates of the benefits of rebalancing strategies. Often tests are conducted over a single sample period. No matter how long the sample period is, its results represent the outcome of a single trial, and it is difficult to assess their reliability in live, future applications.

For example, suppose rebalancing a portfolio to target weights at different calendar intervals (*e.g.*, monthly, quarterly, annually, etc.) is simulated with historical monthly returns for a 20-year period. One rebalancing frequency will yield the highest compound return. How confident should someone be that this top sample frequency also will be the top frequency in the future? For that matter, how confident should one be that the leading rebalancing frequency in one sample also would be the leading frequency for the same target portfolio in an independent historical sample? How confident should one be that the top rebalancing frequency for one target portfolio also would be the top frequency for a different target portfolio even within the same sample period?

Such questions about the reliability or future applicability of experimental results cannot be answered by a single trial or repeated trials with the same data sample. Repeated trials with independent samples are required to obtain estimates of the probabilities that experimental findings are robust. Data availability often limits opportunities for multiple experiments with independent samples. In such cases,

bootstrapping may provide a means of generating information about probabilities from a single sample of historical data.[17]

Repeated experiments with the same data sample are especially unreliable. They often degenerate into brute data-mining exercises. After scores of trials over the same sample period, the finding that a particular set of rebalancing "triggers" (*e.g.*, sell at target weight plus 8 percentage points and buy at target weight minus 3 percentage points) maximizes compound return is of questionable worth. The result is almost certainly time-period specific, and application of the best in-sample triggers going forward in real time is unlikely to yield an increase in compound return of similar magnitude.

Rebalancing simulations frequently ignore costs and taxes. Alternative strategies must be evaluated on a net-of-cost, after-tax basis. At a minimum, simulations of alternative rebalancing strategies should provide measures of the turnover generated by each strategy. Costs usually increase with turnover so it is not obvious that a strategy with higher compound return and higher turnover is superior net of costs to another strategy with lower compound return and lower turnover. But turnover is only a proxy for costs. Deductions of estimated costs and taxes are required to obtain more precise estimates of differences in net returns.

Comparing the results of rebalancing simulations is complicated by differences in risk. Over long periods of time, never rebalancing a portfolio usually will yield a higher gross compound return than any rebalancing strategy. Because never rebalancing entails no costs, it is even more likely to dominate in terms of net compound return. But never rebalancing allows a portfolio to drift far from the target weights, and the risk of an uncontrolled portfolio is likely to be greater on average than that of a rebalanced portfolio. Rebalancing strategies must be compared on a net-of-cost, risk-adjusted basis. Risk adjustment requires a valid measure of risk, and the measurement of risk remains a thorny, unresolved issue.

Properly conducted simulations can yield valuable information for evaluating alternative approaches to rebalancing. However, a major danger of simulations is the risk of self-delusion. No matter how carefully simulations are conducted, a financial advisor should not expect to add scores of basis points to net compound returns by skillful rebalancing.

---

[17] In Clark (1999), I discussed bootstrapping and its application in the evaluation of rebalancing strategies. That paper compared the results of rebalancing over a single historical sample period with those from repeated experiments with bootstrapped data generated from the same historical sample.

Conclusions

Rebalancing decisions are complex. Rebalancing costs differ across taxable accounts. The benefits of rebalancing are difficult to measure, and they differ from investor to investor due to their differing appetites for risk. For these reasons, simple "one-size-fits-all" rebalancing rules applied to all accounts cannot be in the best interests of all clients. For taxable accounts, the potential for harm caused by excessive rebalancing is great.

Knowledgeable financial advisors recognize these complexities. They strive to make rebalancing decisions for each client based on an assessment of that client's unique schedules of costs and benefits. For their extra efforts, financial advisors should charge more for account management.

In their approaches to rebalancing, investment professionals should be guided by four admonitions:

- Do no harm.
- Don't employ naive, mechanical rebalancing rules.
- Don't rebalance just to appear to be doing something.
- Don't promise a lot from rebalancing, and your clients won't be disappointed.

References and Related Papers

Booth, D. and E. Fama, “Diversification Returns and Asset Contributions.” *Financial Analysts Journal* 48, No.3 (1992), 26-32.

Clark, T., “Efficient Portfolio Rebalancing.” Dimensional Fund Advisors, Inc., January 1999.

Constantinides, G., “Multiperiod Consumption and Investment Behavior with Convex Transactions Costs.” *Management Science* 25 (1979), 1127-37.

Constantinides, G., “Optimal Stock Trading with Personal Taxes.” *Journal of Financial Economics* 13, 65-89.

Constantinides, G., “Capital Market Equilibrium with Transactions Costs.” *Journal of Political Economy* (1984), 842-62.

Davis, M. and A. Norman, “Portfolio Selection with Transaction Costs.” *Mathematics of Operations Research* 15, 676-713.

Fama, E. and K. French, “The Cross-Section of Expected Stock Returns.” *Journal of Finance* 47 (1992), 427-65.

Leland, H., "Option Pricing and Replication with Transactions Costs." *Journal of Finance* (1985) 40, 1283-1302.

Leland, H., “Optimal Portfolio Management with Transactions Costs and Capital Gains Taxes.” Haas School of Business Working Paper. University of California, Berkeley. December 1999.